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CONSTANTS OF NATURE, CONSTANTS OF ARCHITECTURE AND MATHEMATICAL INTERFACE

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Abstract: Einstein explained to Henri Poincaré that he gave up on mathematics and entered the realm of physics because although he could discern true statements from false, he could not determine which facts were the important ones. Nonetheless, we can also assert that mathematics seems to be the uppermost human creation, pure poetics. Numbers, relation throughout numbers, leads to representation. Furthermore, there seems to be an invisible world that can only be pictured through mathematics and thus we can imagine, we can see. By following this path, we can then state that art and the aesthetic can be deeply challenged at the level of being human, a creation of the highest order, as well as embodying the aesthetic experience as the sensible experience, according to similar lines of enquiry that are offered by mathematics. Modern science put in motion an idea that seems even critical today, that contemporary knowledge and civilization can only be possible by research into the unseen, being it at the microcosmos of cells and atoms or at the macro cosmologic level. Kari Jormakka explains that the presence of geometry (and thus of mathematics in architecture) seems to be a tool, a subjective option of the architect. Certainly, in each case, options regarding geometry enable impressive solutions and working methodologies, but also bound creation. Einstein-Poincaré-Jormakka may lead us to a world of inquiry about the role of mathematics in architecture, in design. Truth, essence, accuracy in architecture as well as scientific validity can be mechanisms to deeply question the nature of architecture is. In general, humans like order, and constants of nature seem magical. However, when algebra surpasses geometry, mathematical realism seems to display much of irrational numbers being Φ - Phi - a single one among an infinity. The use of Phi seems to offer a practical method where the essence of architecture might lie in other places.

Keywords: Einstein; Poincaré; Jormakka; Mathematics; Method.

The sensuous, the measurable and the incommensurable

Werner Heisenberg's (1901-1976) uncertainty principle implies that reality cannot be observed in all its extent and thus cannot be sensed. By another hand we have Benoit Mandelbrot's fractals that may enable us to visualize phenomena such as the Brownian motion. [Fig. 1] We have come to a stage where abstract mathematical devices can 'see' and yet they may be efficient because they have been worked apart from the world we sense, because mathematics found an inner world of itself where it can prosper.

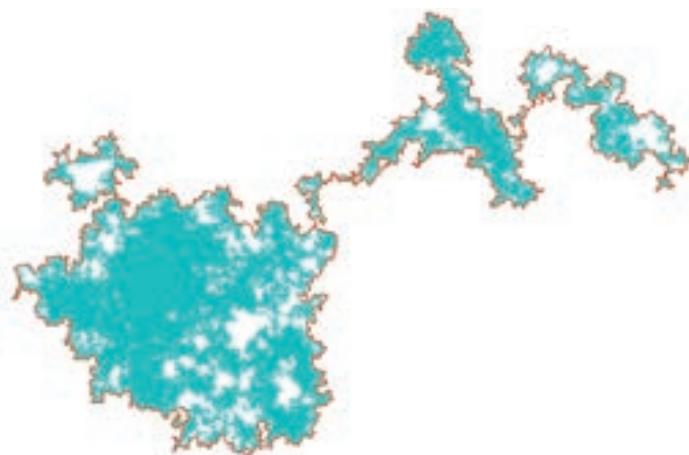


Figure 1. Brownian movement boundary. Fractal dimension = $4/3$.
 Alexis Monnerot-Dumaine © Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Front_mouv_t_brownien.png

The idea of extending our senses through mathematics may seem particularly strange. However, extending our senses through technology might seem more obvious, or friendly. Turning up the volume of the sound to have better hearing and wearing glasses to improve our sight is familiar—but looking through a microscope, a telescope, too. In its own way, riding a horse is an expression of a technology that expands our ability to explore and fashion the world. Riding a horse is also a form of extending human body capability and might have had taken place 3500-5500 BC later than horse domestication about 6000-5500 BC. (Britannica; Equine world UK 2023) Experiencing a horse riding in battle made a huge difference in ancient times and defined and strengthened empires. Also, weapons and tools are common expressions of this necessity of extending our senses through technology and thus there is a long history of human records behind us.

Thinking about what we cannot sense, or measure, seems strange at a first glance and may question our level of certainty. It is the realm of abstraction, whatever might be its links to what we perceive. From microscopes to radio

telescopes visualization seems a main path to our sense of certainty and sense experiences move us. And the vitality of fractals owes much not to a purely mathematical character of its own but to the fact that they display images and can be submitted to an aesthetic judgement. We may say that those images are 'virtual' but nothing is so 'virtual' because an image as such needs a type of materiality to be sensed, to be seen by the eyes.

A characteristic of Dennis Gabor's (1900-1979) holography is that it imitates the way we perceive three-dimensionally, and this is a key to its success. Material and immaterial mingle in this 'virtual' world. [Figs. 2, 3] Besides, by discussing light in terms of a wave or rather a particle in both cases we have something material in our horizon of events. But exploring further some holographic presentations such as the "2019 League of Legends World Championship Finals Opening Ceremony" we realise that even colour is used to strengthen visualization and pure shape geometry. [Fig. 4]

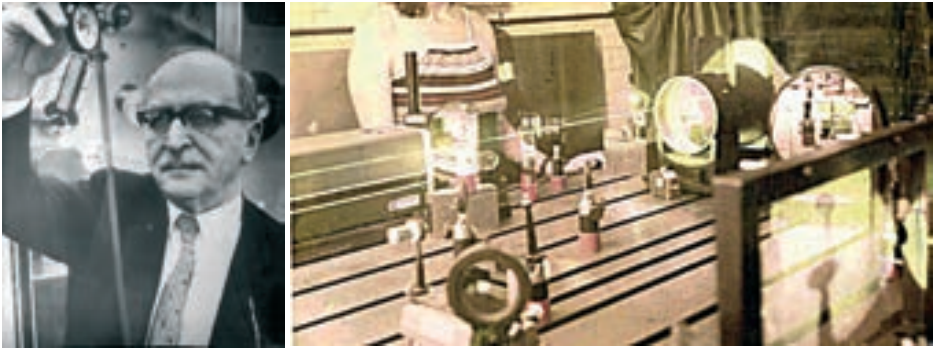


Figure 2. Dr. Dennis Gabor, 1871. Inventor of holography

Associated Press © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Dennis_Gabor_1971.jpg

Figure 3. Holography setup

Epzcow © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Holography_setup_3.jpg



Figure 4. 2019 League of Legends World Championship Finals, Opening Ceremony

G2 Esports © Wikimedia Commons.

https://commons.wikimedia.org/wiki/File:2019_League_of_Legends_World_Championship_Finals_-_Opening_Ceremony.png

Metaphysical beliefs seem to imply physical and thus sense experiences, too. Deities or gods that we cannot see usually imply some type of revelation by ascribing what we sense to an upper entity. For instance, to understand nature as a revelation of the presence of deities implies that we sense both nature and those deities and the former as expression of the latter. But nature itself can be understood as divine, the deity itself.

Turning inside us away from an external world by means of meditation also means a sense experience to which we may assert spiritual meaning. Deities can also assume a material shape, a human shape, and perform miracles which needed to be *meta*-natural, to be *un*-natural and exceptional otherwise they would lose sacred significance.

Prescribing rules for mortals' life on earth accordingly to anticipate a life after dead seems to imply that sense experiences are fundamental to metaphysical inquiry and to the establishment of a theological body of knowledge. In this sense, rules seem to imply measurement because there is a referential that measures by the terms of a judgement. In Christianity, the Ten Commandments are both a referential and a set of measures that prescribe behaviour. Thus, it is a rational system confined to a defined logic that has a metaphysical meaning too.

We may also consider that such system is based on down to earth sense experiences and seems closed, finite, and objective. By another hand, as the metaphysical point of view is vague or, perhaps using a better expression, is incommensurable and thus incommensurability appears to be assumed as an intrinsic essence of transcendence. However, such incommensurability seems to be accepted as a particular sense experience of its own and therefore is perceived. One thing is the absolute that can be measurable and quantified and the other thing an absolute like that we may ascribe to God that would be a metaphysical infinity. With regard to the individual's ontology this was a particularly important problem for which Merleau-Ponty (1908-1961) creates an attractive framework in his "*Le Grand Rationalisme*" (Merleau-Ponty 1996: 179-187), a metaphysical infinity, an absolute that is constituted in and by the individual regarding *himself-to-himself-to-the-world-to-the-others*.

Fashion metaphysics in a world acquainted by the senses

Constructing a temple may express the convergence and synthesis of most complex ideas in which matter meets the immaterial realm of transcendence. But apart from metaphysics, architecture and art need practical means, materials, techniques, and technologies bound by natural laws. The transcendental incommensurability of a deity may differ from the consistence of a temple as a built thing. The simple fact that any construction is finite and measurable means that it will not display any incommensurability except metaphorically, symbolically.

Perhaps a unique expression that does not follow this rule is to be found in Aboriginal spirituality and cosmology. What we may conceive as the 'temple' is given by nature itself, land, water, and sky as territory of events in which

human life happens in a perpetual cycle of being born, living and dying and thus characterized by an infinite space and time. The 'built' thing is nature itself which is embodied as a perpetual space and time dynamic.

With a focus on precise measurable data, we can create an invariable system of proportions and measures that provide us the appropriate tools to build temples everywhere and any time we need. Precise, finite data is thus crucial and easily combined with techniques and technologies to erect buildings. But, besides techniques and technologies, this means that there are finite defined measures for things that grant our access to metaphysical incommensurability.

The phenomenon of mathematics in architecture seems to be clear in some instances such when a *geometrico-mathematical* framework settles down a symbolic system that aims to organize all sense experiences from the physical to the metaphysical and this is the reason why we pay much attention to temples. They seem to be paramount to other types of expression perhaps because besides spiritual constructions they also need to be constructed in matter.

The sensuous, the measurable and the incommensurable may not be exclusive to science and mathematics, not confined to the laboratory where we measure phenomena under control, but rather be rooted on metaphysical beliefs where the divine, nature and we meet and would have different expressions in art and science. The extensive use of expressions such as geometry may combine different types of symbolism and we may question these if the presence of a geometrical framework is confined to myth or to science, or if both mingle.

We would say that infinity itself is ancient to any sense of incommensurability which would be hardly touched by a sense experience. An infinite universe was out there before we could conceive infinity. But it must be stressed that experiencing infinity goes far beyond the huge and that the perception of a crystal-clear night sky displays stars and constellations at a 'finite' distance. As Patrick Heelan (1926-2015) explains, the daily blue sky is often perceived as background to the sun even if we now know that sun is much farther away from our beautiful blue sky. (Heelan 1983: 59) It seems that we experience things on a finite basis, but we have been able to build incommensurability progressively and beyond sense experience. And mathematics seems to have had an important role in acquainting us with infinity. But in so far as we will not discuss here the problem that must exist with infinite types of infinity, one can admit that there may be some way of representing this somehow, beyond Constantin Brancusi's *Endless Column*. [Fig. 5] This column represents a huge challenge in what art can be embodied with mathematics. The crossing point between infinity and a finite material object is a star melting point, it stands at the acme of metaphor and abstraction. In its own way it appeals to a sense experience of incommensurability and perhaps Merleau-Ponty's metaphysical infinity helps us to understand how such perception is possible: an ultimate art-mathematical experience!

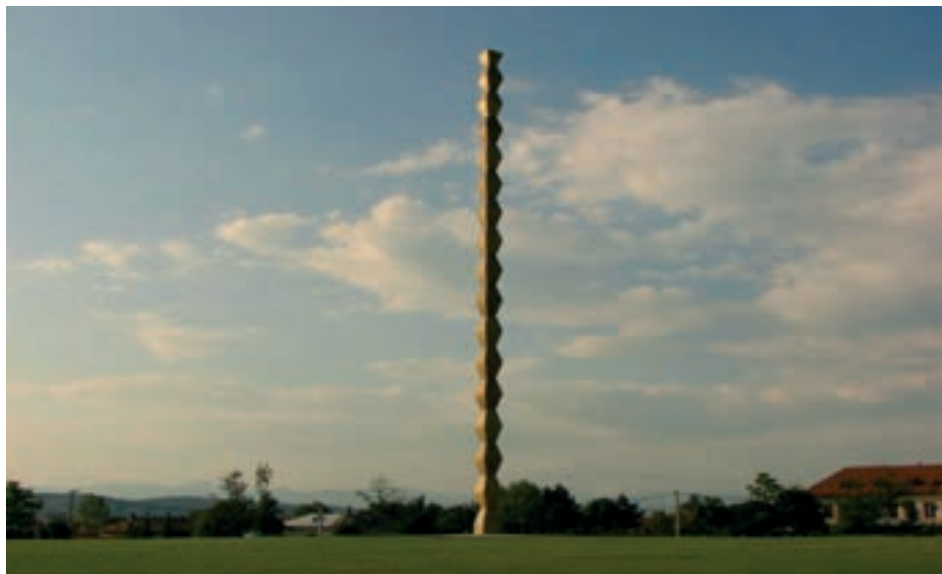


Figure 5. Constantin Brâncuși , “The Endless Column”, Târgu Jiu, Gorj County, Romania, 1938
<https://www.deviantart.com/danniellm/art/Endless-Column-by-C-Brancusi-103404729>

What nature? Which *mimesis*?

Is *mimesis* a ‘natural’ instinct? *Mimesis* is not an autonomous and straight forward concept. By its own sake it depends on nature and thus on conceptions of nature and this is not neutral to the rule of mathematics in art and architecture. Basically, and thinking on an historical basis, we may have one of two different types of nature, the visible and the invisible and these refer to the categories we have already mentioned, the appearances of things and the [hidden] ways in how things work. But we may need to combine them to the dialectics of figurative *versus* abstract. And if we remark that both, figurative and abstract, have different content and meaning, we suddenly see that the complexity of these phenomena grows.

We may depart from using the term *mimesis* as developed from the fifth century on when “the term ‘imitation passed from cult into philosophical terminology and began to designate reproduction of the external world” (Tatarkiewicz 1980: 266) which showed the acquisition of two fundamental meanings and thus we move from Democritus conception of ‘*imitation of the way nature functions*’ to a “*concept of imitation, which acquired greater popularity, was also formed in the fifth century in Athens but by a different group of philosophers: it was first introduced by Socrates and further developed by Plato and Aristotle. To them ‘imitation’ meant the copying of the appearance of things.*” (Tatarkiewicz 1980: 267) But it is also important to remember that, as Władysław Tatarkiewicz (1886-1980) explains, “Imitation was called *μίμησις* in Greek and *imitatio* in Latin: it is the same

term in different languages. The term has existed since antiquity; the concept, however, has changed. Today imitation means more or less the same as copying; in Greece its earliest meaning was quite different.” (Tatarkiewicz 1980: p. 266) But later we are going to use *mimesis* as imitation of reality and thus combining the imitation of nature and the imitation of the works of masters that imitated reality and thus being selective regarding natural features. This idea developed from the Renaissance especially by assuming, according to Vasari, that the artist could create perfect art works in opposition to nature that, sometimes, displayed creations that were not perfect.

To begin with these conceptions, seem important to understand the importance of mathematical expression in western art, painting, sculpture, and architecture but also in music due to the concept of harmony and its mathematical approach by the Pythagoreans which was according to Rudolf Wittkower (1901-1971) was of seminal importance in the architectural theory of Renaissance architecture. (Wittkower 1988) What emerges here is a double manifold concept of nature in which we have a visible world we fashion and a hidden world, where the actual forces of nature’s work are fundamental to *geometrico-mathematical* approach meaning and deep symbolism. And mimesis is then challenged when one must decide what nature should be imitated either the visible or the invisible, or any way by which we may combine them in a perfect imitation.

According to Tatarkiewicz, “Aristotle not only gave a general definition of the concept of nature, but also perpetuated its diversity. Firstly, as he wrote, “the expression nature refers both to a natural process and to the products of that process”. We see the forces of nature in the growth of grain, but we also include the grain itself in nature. And, secondly, the expression nature, according to Aristotle, refers both to the matter of things and to what he termed their form, their essence, the force that directs nature. Nature in the first sense is in ceaseless change, and in the second, on the contrary, “persists despite changing conditions”. This duality of concept has survived as well into modern speech and into the modern conceptual system: ‘nature’ is used to designate both the visually palpable world and the forces, palpable only to the mind, that we surmise to shape the world.” (Tatarkiewicz 1980: 292)

And Tatarkiewicz proceeds with the evolution of the concept by explaining the way in which Romans acquainted this conception of nature. This conception will be extremely important to the development of western art,

“The Romans translated the Greek *φύσις* as ‘*natura*’, and this Latin equivalent took over the ambiguity of the Greek expression. Thus ‘*natura*’ meant on the one hand the *sum* of visible things, *summa rerum*, but on the other the same thing as *origo rerum* and *lex naturae*, or the principle of generation of natural things, the *force* that had produced them. The Middle Ages retained both meanings of the expression, rendering the ambiguity harmless by adding a different adjective to each and thereby differentiating *natura naturans* and *natura naturata*, that is, creative nature and created nature.” (Tatarkiewicz 1980: 292)

In the Renaissance we may speak about a theory of art that combined a theory of beauty, a cosmology, and a theology. The inquiry made by artist such as Leonardo da Vinci on proportion of faces and the human body concern an articulation between what is seen and unseen in nature and this was of critical importance at the time. But such inquiry was a need that artists felt beyond curiosity itself. They needed to struggle with the value of their work, of their art. Classical heritage kept through Middle Ages gave special attention to philosophy, or poetry, but not those arts that simply imitate and thus were not much creative. According to Tatarkiewicz,

“The Greeks had no terms that might correspond to the terms ‘to create’ and ‘creator’. And it can be said, neither had they need of such terms. The expression ‘to make’ (*notsiv*) sufficed them. Indeed, they did not extend even that to art, or to artists such as painters and sculptors: for these artists do not make new things but merely imitate things that are in nature.” “Will we say of a painter that he makes something?” asked Plato in *The Republic*, and answered: “Certainly not, he merely imitates.” (Resp. 597 D.)” (Tatarkiewicz 1980: 244)

This idea focuses the visual, what is seen and can be copied, imitated. Imitating nature becomes the reference for artistic skills. Thus, the artist seems somehow bounded because he/she cannot go beyond sensory data, unless to attempt to explains things. An invisible nature seems impossible to grasp but an artist may not need to explain anything. However, we already see in Villard de Honnecourt’s drawings an exploration beyond what is unseen. The artist needs to grasp nature beyond what is seen. [Fig. 6] (Wittkower 1988)



Figura 6. Villard de Honnecourt, Page from the Sketchbook, about 1230. O MS. 19093 French Collection, Bibliothèque Nationale, Paris (No. 1104 Library of Saint-Germain-des Prés until c. 1800 © Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Villard_de_Honnecourt_-_Sketchbook_-_36.jpg

But Leonardo da Vinci's drawings approach seen versus unseen of in a different way. Proportions are there, they are visual and displayed and one assumes that he simply needs to make them explicit. [Fig. 7] (Wittkower 1988) This capability displays the artist ability to decodify the secrets of nature such as created by God and thus the *geometico-mathematization* shows to be more accurate than a pure philosophic system that combine *concept-word* under a certain logic but not *concept-image* (using Saussure's terminology as reference). Thus, the artist was able to inquire deeply into the divine creation of human being and nature in a way that theology alone could not. Furthermore, by combining the small other of the observable – and thus measurable – and a largescale cosmologic conception of cosmos, the artist and the architect could display a metaphysical knowledge and, according to build [specially] a temple we needed to fully control all material needs, bricks, stones and mortars, techniques, and technologies. In this sense, this atmosphere also created the modern roots of a belief in the primacy of technology that is still felt in the present as a source of progress.



Figure 7. Leonardo da Vinci (1452-1519). "Male head in profile with proportions"
 Frank Zöllner et Johannes Nathan, Léonard de Vinci, 1452-1519 : L'œuvre graphique, Cologne, Taschen, coll. « Bibliotheca universalis », 2016, 768 p., 19,5x14 cm (ISBN 978-3-8365-5440-4)
 Drawings by Leonardo da Vinci in the Gallerie dell'Accademia (Venice) © Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Leonardo_da_vinci_Male_head_in_profile_with_proportions.jpg

Ancient demand for Mimesis or the humanization of the world we live in

We find representations of nature in primitive art. Representations of animalia and of human beings are found in earlier stages of civilization. There might be earlier revelations of civilizations such as communications and invention of language, however, we only have records of languages when they acquired a written form, and thus expressions of *mimesis* as a source of a sense of humanization are, perhaps, among the farthest ancient revelations of what it is to be human. We may then question the accuracy of representations from the likeness to an actual gesture of an expressionist manifestation that can make what is seen and represented alive for what a representation of an animal combines its life and strength under a specific form and shape. And thus, we are telling about a measurability in which the primitive artist engaged their own body and mind as means of acquaintance and expression.

Indeed, *mimesis* combines both figurative and abstract features and we find such manifestations in the cave paintings of Altamira (Upper Palaeolithic period, 35,000 – 11,000 BP), or in the engravings of Côa valley (Upper Palaeolithic to the final Magdalenian/Epipalaeolithic 22.000 – 8.000 BCE). [Fig. 8] But there seems to be other phenomena related to the visualization of natural figures and those are to be found in cosmology. And it would be hard to distinguish if those representations refer either to external appearances alone or to the mechanisms that make nature work. Most probably such representations may embody both concepts to some extent even if they did not exist at that time. Yet and being aware of the magnitude of time that separates that ancient world to Romanticism, we may say that those early figurative representations might be ascribed to both the sense experiences of the picturesque as to reflective experience of the sublime.



Figure 8. Prehistoric Rock-Art Site of the Côa Valley - Penascosa - Bull (6 August 2011)

Reino Baptista © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Prehistoric_Rock-Art_Site_of_the_C%C3%B4a_Valley_-_Penascosa_-_Bull_@_2011-08-06.jpg

Mimesis-based frameworks enable us to reflect on other kind of human records. Cosmology combines both material and immaterial records. Regarding “The world’s oldest continuous culture for millennia,” as Hetti Perkins says, we have the oral tradition, and the sky is the «natural engraving» where we find the Great Emu [Fig. 9] and the famous Emu engraving is located at the Elvira Track, West Head, north from Sydney. However, thinking about oral tradition, the image itself might be rooted over 50.000 years ago and was kept alive in the stream of time and space and of human culture. Thus, there is a material and a immaterial record combined, the sky and the oral tradition.



Figure 9. The Australian Aboriginal constellation of the “Emu in the Sky”, which stretches from the Coalsack at its head, to Scorpius to the left.

Barnaby Norris and Ray Norris © Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Emu_public.jpg

For the present argument the emphasis goes to the fact that is an image that has a *mimesis-like figurative-abstract* structure rather a *geometrico-mathematical* framework. Such phenomena may give us a sense of early revelation of how a *figurative-imagistic* order can ground our comprehensive understanding and acknowledgment of ourselves and all the world we live in and consequently, we are already thinking of a profound conscious culture.

Another level and expression of cosmology comes already embodied with a mathematical framework or, at least, has a mathematical order on the horizon. Regarding Ptolemy (Latin in full Claudius Ptolemaeus, born c. 100 CE—died c. 170 CE), the Egyptian astronomer, and his work *Hē mathēmatikē syntaxis* (“The Mathematical Collection”) and we see the sense of a mathematical order to be discovered in the heavens. But geometric expression in human records is yet to be found down on Earth and not in celestial imagery.

‘Besides’ *mimesis* there are other records that witness human culture and those might be ancient to any kind of mimesis approach, there are artifacts such as a Flint Cleaver found in Griquatown, Northern Cape Province, South Africa from the Lower Palaeolithic, between 300.000 and 1.000.000 BP. [Fig. 10] We see knowledge of materials and precise techniques, but the overall shape seems from the realm of two and three-dimensional geometry. Indeed, our sense experiences relate to touch, to textures, weights, and temperatures, but geometry is unavoidable and thus to impose a given geometry to a given object like a stone is an early human revelation.

However, as soon as we assume that those shapes require some understanding of how nature works, we then have to assume that, in fact, those revelations are also *mimesis-like* structured action. Then, what we may consider is that different developments of *human-like* actions do not necessarily imply a simultaneous development of different types of *mimesis*. We should keep in mind that we are using later concepts of *mimesis* which are a late occurrence in history to describe ancient human records.

Those early humans certainly looked at the sky, but we need to refer to astral maps, no oral tradition and we hardly imagine an earlier sense of imagery provided by a *mimesis-like* approach. But according to their needs, would they need such records, or would they already have the skills to record them? Thus, what remained from those individuals as human records are themselves – that is their skeletons, their body parts – and tools that we may easily ascribe to geometric related skills that they would imagine creating proper forms and shapes to pursue an end. This is the territory of abstraction and geometry that takes us to, but not only, forms of mathematical conceptualization.

In Altamira-Côa we may not think explicitly about mathematics, but we may not avoid a sense of synthesis of form that retains the major characteristics of different animals, and such synthesis reveals a sense of abstraction, of balance between the figurative and the abstract which are important phenomena to art, science, and mathematics. The thing as such stands side by side with the thing represented and both partake the sense of being. However, Altamira-Côa synthesis of form implies skills in abstraction that are different from those of Griquatown. [Fig. 10] Different objectives and a sense of the awareness of the spiritual may explain the growing of a new sense of conceptual and formal abstraction which *mimesis* may express.



Figure 10. Cleaver, between 1000000 and 300000 BP (Lower Palaeolithic). Griquatown, Northern Cape Province, South Africa. Collection of Edward John Dunn (1844 - 1937).

Museum of Toulouse, Archaeodontosaurus © Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Silex_Griquatown.2009._0.197.1_Noir.jpg

According to the lines of this reasoning we may arrive to a context in which acquaintance with geometry of objects expressed by a capability of creating them, roots mathematic forms of acquaintance with the world on ancestral revelations of being human. Yet we may have no evidence of a myth free acquaintance with the world.

Reality without myth

Thales of Miletus (c. 620 BCE - c. 546 BCE) lived before Plato (c. 428 BCE or 427 BCE - c. 348 BCE or 347 BCE) and Aristotle (384–322 BCE). He “is often cited as ‘the first philosopher’, because he is the first person known to have asked and answered a question about nature and source of reality without recourse to myth.” (Grayling 2021: 3) And he found the *cosmos arche* (‘principle’) being *water*. (Popper 2002: 87, 106) In some sense, this is a form of *geometrico-mathematization* in which *water* is framed as an *algebra-free* constant of nature, somehow, equivalent to an occurrence at the Periodic Table of Elements or even to an algebraic expression of a natural law.

Then, if we think on the icosahedron [Fig. 11] as symbol of water we have created an immutable object that behaves like a constant of nature, namely it is permanent whatever water is solid, liquid or vapour. We may then verify that the icosahedron is not time orientated and therefore is timeless and time-free. This sense of absolute may not stop here if combined with other characteristics of a Euclidean knowledge-based framework. Furthermore, we have geometry playing an important role in acquaintance with nature and cosmos. Geometry simultaneously inquires, questions, and answers, explains and makes such explanation visual and thus obvious. That is, geometry returns to visual sense experience, a complete set of logical scientific operations that an algebra alone cannot. The demand for the visual is crucial, to see is to know.



Figure 11. Luca Pacioli giving a lesson in geometry to a pupil. Portrait by Jacopo de' Barbari (1440-1515).

“One of the Platonic solids, a dodecahedron, is seen on the right resting on the top of Pacioli’s book *Summa*. Pacioli himself, dressed in friar robes and almost resembling a geometrical solid, is shown copying a diagram from volume XIII of Euclid’s *Elements*. A transparent polyhedron known as rhombicuboctahedron (one of the Archimedean Solids, with twenty-six faces of which eighteen are squares and eight equilateral triangles), half filled with water and hanging in mid-air, symbolizes the purity and timelessness of mathematics.” (Livio 2002: 130)

Galleria Nazionale Di Capodimonte in Naples, Web Gallery of Art © Wikimedia Commons
https://commons.wikimedia.org/wiki/File:Jacopo_de%27_Barbari_-_Portrait_of_Fra_Luca_Pacioli_and_an_Unknown_Young_Man_-_WGA1269.jpg

Under this construction, the icosahedron would be the most ‘fundamental particle’ of nature. However, it would be hardly flattened two dimensionally creating some problems to one who builds walls and vaults where an empirical constructed form plays its role, too. Despite the tridimensionality of architectural construction, systems of proportions usually display, or depart from two-dimensional geometric image systems. In this sense, they are very selective. Making a remark, such systems may hide the tectonics of the building and by necessity they are confined to tectonics.

When we say that in the Renaissance theory of art that combined a theory of beauty, a cosmology, and a theology, we add that artists were especially concerned with truth beyond myth. Mimesis symbolises truth at the core of the engagement of the artist with his creation. In this sense it is a scientific method and thus free from both falsehood and poetry. It is objective. Thus, Luca Pacioli (c. 1447-1517) was providing the adequate principia to ground art in truth. [Fig. 11]

Mimesis had not played an obvious role in architecture like that of painting and sculpture since it implied some type of figurative expression. Leonardo’s drawings would express the crossroads of *summa rerum* and *origo rerum*, but the answer was objectively figurative and using the concept of form as the contour of the object, and thus making such statements more precise and somehow equivalent to those lines that define pure geometric figures. Buildings do not grow naturally in nature and the shapes of trees, flowers and animals may not be suitable for building. Marc-Antoine Laugier’s (1713-1769) primitive hut is already a selective approach organized by a tectonic structure made clear by a geometry. [Fig. 12]



Figure 12. Charles Dominique Joseph Eisen French (1720-1778). Frontispiece of Marc-Antoine Laugier’s *Essai sur l’Architecture* 2nd edition, 1755. Allegorical engraving of the Vitruvian primitive hut.

Charles-Dominique-Joseph Eisen © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Essai_sur_l%27Architecture_-_Frontispiece.jpg

Geometrico-mathematization by displaying geometric arrangements of propositions given by accurate commensurable measurements represents the *modus operandi* of nature, the hidden working forces of nature that combine a tectonic truth – and therefore the construction is possible as such – and the inner secrets that are omni present all over and comprehensibly throughout all forms and shapes that are to be found in *cosmos*. In fact, the role of geometry is to work out both art and reality without myth.

What we see is not what we can create, and imitations are ‘imperfect’

Heisenberg identified operational impossibilities on approaching the intimate core of phenomena and yet, we may find similar problems in the ancient world when the capability to inquire into reality without myth was already conscious and objective. The ancients recognized that the problem of observation, the problem of the visible and its reproduction was not innocuous when we explore authenticity and truth. The necessity of objective observations was there, “However, the situation was by no means simple. The ancients realised that even if we do not alter reality intentionally, we do so unintentionally; our eyes change – deform – what they see. We see a man differently close up than at a distance, differently in the sun than in the shade; and obviously, he cannot be simultaneously this way and that. Thus arose the question, how the painter and sculptor are to represent the man - as he is, or as we see him? Greek scenographers and skiagraphers painted as we see, convinced as they were that that is the proper way truly to represent things. Philosophers of that time, Democritus and Anaxagoras, studied the laws of perspective in accordance with which we deform the things seen; they felt that the same laws should be applied in art. However, Plato did not interpret the truth in this way; to him every deformation, even one in accordance with the laws of optics, was a falsehood.” (Tatarkiewicz 1980: 301)

A solution to this logico-conceptual problem came from a scholar of the ancient world, archbishop of Seville, “Isidor of Seville (c. 560 – 636) had separated ‘falsum’ from ‘fictum’: Art comprises not truth and lie, truth and falsehood, but truth and fiction. Yet most mediaeval and Renaissance theoreticians did not remember this and continued to measure off the truth and the falsehood in art.” (Tatarkiewicz 1980: 302)

In order to express truth, we may have to consider that in this context where we do not evaluate properly the importance of settling down truth and fiction as characteristic to art, we have found a subsequent over evaluation of the importance of mathematics at the core of art. Therefore, mathematics does not arise from its inner strength as a body of knowledge itself, but rather from an extrinsic input, a subjective option that engages art and science but, nonetheless, it is a fallacy in which myth is confined to reality. Yet, we need to keep in mind of the importance of *geometrico-mathematization* in art. The geometric arrangements found on the organization of parts of the medieval cathedrals and churches as well as in the Renaissance is grounded in this search for truth. But they are

master pieces for other reasons and not the geometrization of this kind. The idea expressed by a total artwork – *Gesamtkunstwerk* – does not emerge from a geometrico-mathematization alone and the materialization and spatialization of an idea such as *lux continua* and *lux mirabilis* takes over a mathematical framework alone. [Norberg-Schulz 1980: 98, 100] [Fig. 13]



Figure 13. Sainte-Chapelle (Upper chapel), Paris, 1243-1248. Masterpiece of Gothic Rayonnant style and an example of a total art work, a *Gesamtkunstwerk*.

Cmcmm1 © Wikimedia Commons

[https://commons.wikimedia.org/wiki/File:Paris,_Sainte-Chapelle,_Obere_Kapelle,_Innenansicht_\(29\).jpg](https://commons.wikimedia.org/wiki/File:Paris,_Sainte-Chapelle,_Obere_Kapelle,_Innenansicht_(29).jpg)

This desire for truth displayed in art works had consequences in art but they are not necessarily the same due to what “space” is in architecture, sculpture, and painting. Erwin Panofsky makes clear the artificial space of perspective given by a *quantum continuum* that is alien to the real world. However, progressively, the space grew in complexity through an awareness of time and motion through space. Wittkower elucidates this phenomenon by the creation of new rhythms defining the organization of the orders such as in Andrea Palladio’s Il Redentore, Rio della Croce, Venice, 1576-1592. (Wittkower 1988) [Figs. 14-16]

Subsequent developments in cosmology and mathematics in which according to Gideon we will find Borromini (1599-1667), Pascal (1623-1662), Newton [1642-1727] as representative figures of an age, will imply the reformulation on the notion of space recognizing a relativist consistency. However, time remained absolute according to Newton, and in what opposed to Leibniz (1646-1716) to whom both space and time were relative. Using the help of optics, lenses and mirrors, Baroque painters were able to construct architectural spaces in which

relativist space and time were sensed even if not consciously conceived by evaluating the relativist essence of time. Yet they clearly emphasised that the changing of position is space and only motion makes such a perception possible.

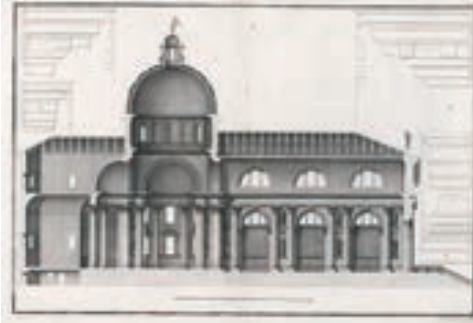


Figure 14. Andrea Palladio. Chiesa del Redentore in Rio della Croce, Venice, 1576-1592.

Drawing by Ottavio Bertotti Scamozzi, 1783. Rhythms created by intercolumniation

Ottavio Bertotti Scamozzi © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Chiesa_Redentore_sezione_1_Bertotti_Scamozzi_1783.jpg

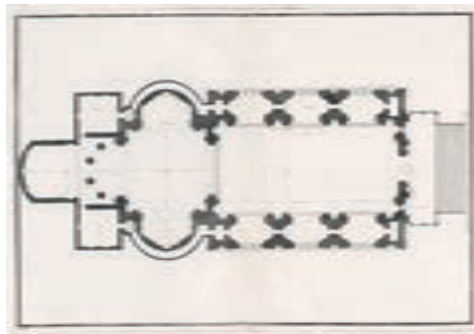


Figure 15. Andrea Palladio. Chiesa del Redentore in Rio della Croce, Venice, 1576-1592.

Drawing by Ottavio Bertotti Scamozzi, 1783. Rhythms created by intercolumniation

Ottavio Bertotti Scamozzi © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Chiesa_Redentore_pianta_Bertotti_Scamozzi_1783.jpg



Figure 16. Andrea Palladio. Chiesa del Redentore in Rio della Croce, Venice, 1576-1592. Interior view.

Didier Descouens © Wikimedia Commons

[https://commons.wikimedia.org/wiki/File:Chiesa_del_Redentore_\(Venice\)_Interior.jpg](https://commons.wikimedia.org/wiki/File:Chiesa_del_Redentore_(Venice)_Interior.jpg)

The presence of *geometrico-mathematic* formulation in space is to be found from the abandonment of the circle as a key stone to cosmology and by its replacement by Kepler's ellipses and the axis would be confined to physical properties of the heavens such as gravity that is as invisible as it also extends beyond our sense experience. Thus, mathematics acts by extending our perception through abstract devices, but André Le Nôtre's (1613-1700) "infinite axis" of Le Vau-Le Vicomte (1656-61) or Versailles (1661-87) will give a material, visual shape to how infinity can be sensed in a finite world.

Geometry and algebra combined seem to overcome all imperfections that imitation might have provided simultaneously and provide access to the intimate essence of art and to scientific accuracy. But how to express it in art terms at the level of the artwork? Tiepolo's space will characteristically assume an essence of being infinite and relativist created by a plastic chronology of space-time through the pictorial space and thus, illustrates both a sense and an objective experience. The latter equivalent to that provided by science. And we may poetically say that it is from and by Tiepolo's artwork that Isidor of Seville's *truth* and *fiction* have not been lost but acquainted. (Giambattista Tiepolo 1696-1770) [Fig. 17]



Figure 17. Giambattista Tiepolo. Gloria of Spain. Salón del Trono, Palacio Real de Madrid, 1762-1764. https://commons.wikimedia.org/wiki/File:Tiepolo_-_Sal%C3%B3n_del_Trono,_Palacio_Real_de_Madrid.jpg

Mathematical origins of architecture

It seems extremely difficult to find evidence and to demonstrate that some type of mathematical thinking, or need, lies in the roots of architecture. However, any construction must obey the laws of two and three-dimensional flat space that can be characterized by geometrical means. Besides material and its placement as a construction, it must follow the laws of physics such as gravity, attrition, resistance to compression and traction, or the mechanics of surfaces. We may also add chemical compatibilities between organic and mineral based materials.

Expression of mathematics in architecture by the means of a geometry or *geometrico-mathematization* seems to find its strength on intellection and not so much on direct sensory experience or on necessity of shelter. Even when we emphasize that primitive forms of acquaintance with materials, forms and shapes have created and built a stream of mathematical abstraction extending that idea *mutatis mutandis* to architecture might not be obvious. In many senses, there is a gap and fundamental difference between the shape of the tool that cuts the stone and the final shape of the that stone. We hardly produce a parallelepiped stone block by using a similar block to cut it. But this idea might be questionable to some extent in relation to primitive stone tools. One stone breaks the other to create a specific shape – like a blade – and yet we may consider that such technique implies a selective approach to stones regarding hardness, size and characteristics of fracture.

When we examine Egypt, Babylonia, Greece, Rome we are already able to find advanced stages of civilization in many fields such as science, mathematics and architecture and knowledge mingles in different ways. Thus, elements such as Greek or Egyptian proportions are fascinating. The problem really goes back to the dawn of Man and civilization and in the ability to appropriate space even if we can claim a natural acquaintance to two and three-dimensional space. But to be submitted to the unbreakable laws of geometry and to use them in a creative way, to fulfil an end, looks entirely different.

This process seems even harder when we think on natural shelters that were not actually built but appropriated and internally ‘organized’ to make life possible. People would eat and sleep somewhere under some special organization. But we would have the natural properties of flat space imposing its characteristics which would not necessarily imply a level of abstraction beyond acquaintance with space.

We may think that, for instance, people around fire may have built a circle. However, we may need to take into account that a fire tends to emit heat approximately on a circular way – in fact by heating the air the vertical expansion is not spherical but if people sit down around a circle we may consider that sense perception of heat, created a regular roundness that people would appropriate by finding themselves at a similar distance given by what the body will consider a comfortable distance. But, in fact, there are the natural laws of flat space geometry and heat expanding along that space that determines that creates the arrangement. Perhaps we can then say that the geometrical characteristics of flat space determine all events in a way that seem to be perceived by the terms of a geometry.

This problem is important in existential and phenomenological terms when we argue that all forms of *geometrico-mathematization* emanate from perception, from sense experience, and *geometrico-mathematization* already is an advanced form of reflection. According to this line of thought we may think that basic acquaintance with the world we live in may be read as characteristically mathematical by the means of a geometry rather than by any form of algebra in which counting should have been an early manifestation of an algebra-like

structure. We may also consider differences in perceiving and creating tools and space for living in, due to the way in which our body and mind are involved in these processes. But the most important would be that we have been able to place a *geometrico-mathematical ability* – or phenomena – at the origins of architecture. Thus, at the dawn of acquaintance with space, sensory space and *geometrico-mathematization* of that space would not have been separated, but separable, and thus both could have been combined further on. This process suggests a unity of *being-and-the-thing* starting at the core of perception in which sensory experience and *geometrico-mathematization* are two faces of the same coin. This is not neutral to phenomenology because it might imply that an *eidetic* reduction might, or might not, have a *geometrico-mathematization* expression of some kind on the importance of mathematics and geometry to full embodiment with phenomena.

Astro geometry

“On the winter solstice, the longest night and shortest day of the year, a shaft of light enters a perfectly positioned window and lights up the sixty-foot corridor that leads to the basin of ancestor bones, an impressive feat of Engineering considering its Stone Age builders possessed only rudimentary technologies.” of the Tomb of Newgrange in Ireland. (National Geographic, UNESCO) [Figs. 18-19] Structures such as Stonehenge (built between 3000 and 1520 BCE), this 5000 years old construction is a typical form of the presence of astro geometry. Being hard to know how those people made and combined astral records of the motion of astral bodies such as sun, moon and night stars, we may not have evidence of observations recorded under an algebra-like structure and thus what usually springs out is the capability of working in terms of a flat geometry to understand natural events and record them.



Figure 18. Tomb of Newgrange, Ireland. Brú na Bóinne - Archaeological Ensemble of the Bend of the Boyne

Shadowgate © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Newgrange_14.jpg

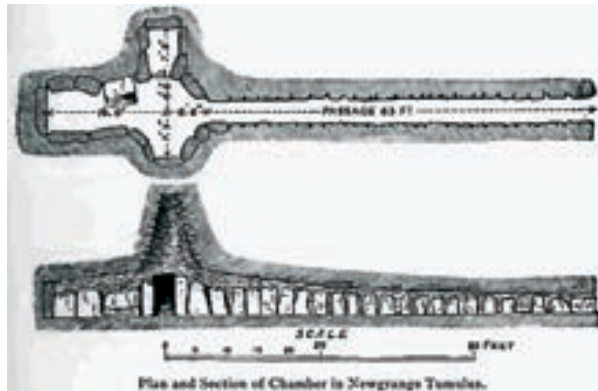


Figure 19. “Plan and Section of Chamber in Newgrange Tumulus” by William Frederick Wakeman, 1903.

Brú na Bóinne - Archaeological Ensemble of the Bend of the Boyne

William Frederick Wakeman © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Wakeman_Newgrange_tumulus_chamber_cross_section.png

We are used to the idea of measure to be given by number. But a sense acquaintance with nature may need to be countable and thus may be performed by other types of measurement. This seems to be important because to think about mathematics in architecture is pretty much biased by the classical tradition. We may think on Greek mathematics as being, indeed, mainly geometric in opposition to the Babylonian being algebraic and yet what comes to our mind are the irrational numbers they were working with, Phi and Pi. And we read Egyptian pyramids according to these lines. That is, we can question the accuracy of our reading of the way they were thinking in terms of a mental process in opposition to the final built object we can read and measure.

Remembering the seminal book of Wilbur Richard Knorr, *The Ancient Tradition of Geometric Problems*, we may say metaphorically that the geometric tradition was created so long as it was seen through solving problems. But it was the way in which the Greeks transformed earlier algebra and geometry that have influenced mostly western civilization from art thought science and some aspects of these phenomena were discussed above in this text.

Body painting and topological geometry

A great difficulty of combining different phenomena comes from specialization and is nonetheless inevitable. Science needs and works with specialization. In this environment where we travel from the general to the singular it often seems difficult to settle down somewhere between those distant realities. Besides, we may even question the way in which we can ‘travel’ from one side to the other without a biased reasoning and thus concluding towards useless data. But we may take the risk.

We find in Aboriginal culture the oral history of astronomy and some records such as the “Emu in the Sky”, but we also find body painting such as we find in many other ancient cultures, Māori in New Zealand, or the great diversity of Amazonia and Africa. Body culture of these diverse continents, that involves tattoos and cosmetics has evolved over centuries and is spread worldwide. However, ‘primitive’ body paintings are more interesting in the context of the present argument.

In general, ancient body painting implies emphasis and is expressed by a topological conscience of the body as being a complex surface. A line by itself already gives a sense of geometry, a line is selective. In order to follow the body surface, face, trunk, harms, hands, feet, lines usually emphasise features of the body and ‘progressively’, we may find shapes of a more geometric character along the topological body. Thus, the body seems to be the convergence and outcome of all arguments that combine acquaintance with a form of topological geometry and the two and three-dimensionality of the space we live in.

We may say that the surface of land we stand on curves, but the surface of our skin is a basic and fundamental source of contact with the world, an important source of sensory experiences. Perhaps, the geometric tradition arises from the experience of the body itself and being an object among objects where all may be acquainted by and in terms of a geometry. If it is to be so, then, *geometrico-mathematization* is far more closely related to perception than it seems. And we may need to reflect on Husserl’s *Krisis (Die Krisis der europäischen Wissenschaften und die transzendente Phänomenologie: Eine Einleitung in die phänomenologische Philosophie)*, and phenomenology in general on another way because there can exist forms of *geometrico-mathematization* that may imply forms of geometry even if they have not yet been undertaken by an algebraic shape.

The problem of body painting and representing acquaintance with the space we live in leaves a door open to the mathematic origins of architecture. As far as consciousness of being implies some sense of geometry, then, all gesture towards creating a living space, or a shelter should imply a form of geometry where the properties of space as such impose their characteristics.

Body painting and Riemannian geometry

“Lobachewsky and Bolyai, two mathematicians working independently of each other, developed a system of geometry which was based on a contrary of Euclid’s parallel postulate”. (Nagel 1979: 237) Lobachewsky -Bolyai pure geometry symbolises the beginning of a new era for mathematics and physics, and it is also arguable that for architecture as well because contemporarily geometry, for example, using algorithms, would not be possible without this revolution and it has shown to be attractive among architects. Combining human body and Riemannian geometry seems challenging and interesting. Leonardo’s concept of form was that of the Reminiscence, for as contour of the object. This idea showed to be efficient on analysing the human body in terms of constituent parts as well as on depicting features of those parts. And, by another hand, platonic

solids are made of points and lines that define surfaces and the latter volumes. Consequently, form as contour of the object was effective across art and science.

Major symptoms of a change came under Tiepolo's brush by representing – especially – clouds and sky where nuances of surfaces overcame a sense of contour. These ideas would be particularly fruitful for Cézanne, Vincent Van Gogh, and the impressionists where a combination of a sense of surface and effect of the object was not produced by that former sense of contour which is “naturally” given by the description based on Euclidean geometry. Indeed, there is a hierarchy in the Euclidean system, *point-line-surface-volume*, that limits our sense of thinking in terms of surfaces. Due to this hierarchy, we are confined to a small number of possible rationalizable surfaces in addition to the lines that can be there and to the properties of those lines. We can metaphorically say that the sphere was then a miracle and the ellipsoid a meta-miracle.

According to Nagel, “The Lobachewsky-Bolyai pure geometry is not the only alternative to Euclid. For the Euclidean parallel postulate can be replaced by a contrary that is different from the one adopted in the Lobachewskian system. A distinctive non-Euclidean geometry is the fact obtained if the Euclidean postulate is replaced by the assumption that through a point outside a given straight line there are no parallels to the latter. In this case, however, other Euclidean postulates must also be amended, for example, the postulates that a straight line can be prolonged indefinitely, and that two points always determine a unique line. The pure geometry obtained when these alterations are made is called “Riemannian,” although Riemann arrived at it by developing Gauss's notions of curvature and geodesic rather than by using axiomatic method.” (Nagel 1979: 240)

Anne Paso's *Lovi Bird* displays lines that define surfaces and the latter volumes once mounted according to a rational order [Fig. 20], and yet we may ascribe this rationality to a spherical geometry. Thus, the birth body would be stretched, and the “meridians” would meet in both opposite sides, beak, and tail, the two “poles” of bird geometry. The originality of the design thus springs from a traditional Euclidean conception to be rejected for another way of seeing objects in space. And this may be one of the characteristics of the design – the only to be focused on the present argument. The geodesics of the Euclidian plane seems short to describe the bird. To be precise, the lines on the geometry of the flat plane only describe efficiently each one of the mounting parts of the bird. Thus, the conception of the bird combines two different geometric systems and emphasises both.



Figure 20. Anne Paso's *Lovi Bird*
Photography © Joaquim Santos

However, flashing back to traditional body painting in some ancient cultures that are alive today we find similar approaches to the body regarding the face, the head, the trunk, arms and legs. Body cannot be flattened and our sense of being in a body informs us of that property of living in a three-dimensional world.

Māori body painting also transmitted to masks shows the use of the body surface that is not particularly Euclidean but rather topological or Riemannian-like. Lines emphasize body lines. The warrior lives from that emphasis. [Fig. 21] Body painting of the Karo Tribe, Ethiopia, display similar characteristics regarding geometry. [Fig. 22]



Figure 21. Maori chief, Anehana, poses against a plain background. He has a thin white beard, short, white hair decorated with a feather.

Depicted place: Longfellow House - Washington's Headquarters National Historic Site, Middlesex County, Massachusetts.

Longfellow House © Wikimedia Commons

[https://commons.wikimedia.org/wiki/File:Maori_chief,_Anehana,_poses_against_a_plain_background._He_has_a_thin_white_beard,_short,_white_hair_decorated_with_a_feather_\(387ce1f9-322d-4ca6-99c6-901787af63e0\).JPG](https://commons.wikimedia.org/wiki/File:Maori_chief,_Anehana,_poses_against_a_plain_background._He_has_a_thin_white_beard,_short,_white_hair_decorated_with_a_feather_(387ce1f9-322d-4ca6-99c6-901787af63e0).JPG)

Figure 22. Portraits Black & White Ethiopia

Rod Waddington © Wikimedia Commons

[https://commons.wikimedia.org/wiki/File:Soothsayer,_Karo_Tribe,_Ethiopia_\(21252291038\).jpg](https://commons.wikimedia.org/wiki/File:Soothsayer,_Karo_Tribe,_Ethiopia_(21252291038).jpg)

We have criticized some aspects of nature, the geometry of nature and ways of perception of acquaintance. Having this background, we may be confronted with the possibility that elementary acquaintance with the body, by stressing its form and shapes using geometry is much closer to that geometry than modern physics requires. However, we have the western tradition of geometry. Ernest Nagel explains an interesting problem regarding “The Choice of Geometry”,

“Given a certain class of spatial properties and relations of bodies, which *language* shall we use to formulate them, and what reason is there for preferring one language to another?” the answer is obvious. “As far as the empirical facts to be codified and predicted

are concerned,” we are compelled to reply, “it will make not an iota of difference which language we adopt. However, we may find one language *more convenient* than another, perhaps for several reasons. For example, we may find the Euclidean language psychologically simpler than the others, if only because we are more accustomed to it.” (Nagel 1979: 253-254)

We may observe that Māori body painting motifs can be “flattened” when we see clothing and architecture. [Figs. 21, 23,24] A demonstration that all types of ornament have the same origin may be questionable but seems plausible to accept all our engagement with the environment starting in-us-from-us and thus we may accept these observations as elements to be explored. What we may think is that in a culture that was not “biased”, that works on different grounds from the western geometric tradition we may find impressive flexibility in the use of different types of geometry.



Figure 23. Maori Pare - Lintel from a Maori Meeting House
Canterbury Museum, Christchurch - Joy of Museums
Gordon Makryllos © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Canterbury_Museum,_Christchurch_-_Joy_of_Museums_-_Maori_Pare_-_Lintel_from_a_Maori_Meeting_House.jpg

We could speculate that the western necessity of discovering a constant of nature created a necessity that would have prevented a flexible use of geometries such as found in Māori culture, and we may go on to sub-Saharan African cultures and Pre-Columbian cultures.



Figure 24. Maori Pātaka or Storehouse Panels
Canterbury Museum, Christchurch - Joy of Museums
Gordon Makryllos © Wikimedia Commons

File:Canterbury Museum, Christchurch - Joy of Museums - Maori Pātaka or Storehouse Panels.jpg - Wikimedia Commons

West in nest

The torus was particularly studied in the Renaissance, a geometric solid represented in an intarsia panel by Fra Giovanni da Verona at the Palazzo Ducale di Urbino where we see the reading of the “donut” according to a constructive system of small parts – flat surfaces – put together finishing the three-dimensional geometric puzzle. [Fig. 25] Euclidean geometry shows its capacity to solve three dimensional complex forms beyond those given by the Platonic solids. But using a non-Euclidean geometry, lines across the torus surface show different properties of those defined by Euclid’s axioms and definitions. [Fig. 26] There is a whole realm of characteristics that we could not have found once we were bound by the Euclidean universe.



Figure 25. Fra Giovanni da Verona. Coro intarsiato di monte oliveto maggiore, 1503-05.

Intarsia panel by Fra Giovanni da Verona at the Palazzo Ducale di Urbino.

Sailko © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Fra_giovanni_da_verona,_coro_intarsiato_di_monte_oliveto_maggiore,_1503-05,_12.JPG

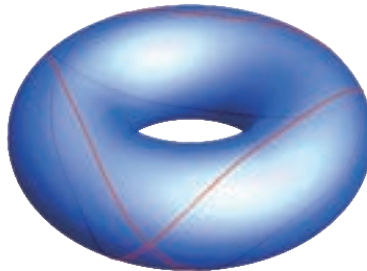


Figure 26. Torus closed geodesic.

Rectas © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Torus_closed_geodesic_n3c.png

Let us flash back to the realm of nature. Bird tree nests are clearly strong structures and yet the weave resembles more a topological or Riemannian geometry exploration of structural possibilities rather than Euclidean properties which are common in basketry, fabric manufacturing and in many traditional crafts. [Fig. 27]



Figure 27. American Robin Nest with Eggs.

Laslovarga © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:American_Robin_Nest_with_Eggs.jpg

Different cultures around the world have created and use heads rings to carry weights such as baskets and ceramic pots, for goods and water. From banana leaves to wool and cotton fabrics they present the common donut-like shape. In order to give some structural finishing, the outer layers often present a Euclidean-like approach to the torus form. Basically, there are two main constructive parts, the inner part goes along a circular path creating the main thickness of “donut” and a second goes across this first layer and is made of stripes where the weaving can be decorative, too, and thus showing the “tectonics” of the form that puts all elements and layers together. This idea of crossing constructive elements seams particularly Euclidean. [Fig. 28]



Figure 28. Ring head to carry weights/objects. Portuguese.

Craft artist: Idália Francisco © Rodilhas da Minha Aldeia, Pampilhosa da Serra.

Photography © Joaquim Santos

However, regarding the duck toy from ancient Egypt and the way in which major constitutive elements are put together they resemble more a topological, or Riemann reading of the form despite the existence of the same elements made across others. [Fig. 29] We may suggest that the choice of a suitable geometry in pursuit of an end also proceeds from natural familiarity with materials and their properties, such as thickness, malleability, perception of the mechanics of surfaces, or structural stability. The importance of this reflection is to question the western geometric malleability and to examine whether there are other forms of geometry that oppose the Euclidean and that may be perceived intuitively and self-evident in different contexts.



Figure 29. Papyrus Toy Duck Roman Period Egypt Oxyrhynchus at the Penn Museum, 50 BCE-400 CE. Egyptian antiquities in the University of Pennsylvania Museum.

Mary Harrsch © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Papyrus_Toy_Duck_Roman_Period_Egypt_Oxyrhynchus_Penn_Museum.jpg

This formulation has fundamental implications for the universality of systems of proportion that have used irrational numbers that are – or at least symbolise – constants of nature. Thus, despite of the importance of those systems, we may question what role they have had and what role we should give to them.

Right here there is the problem of interpretation that does not come from a scientific basis. What value should we give to a particular *geometrico-mathematization*? Is it the *foundation-stone* of the building where all arguments originate? Or is it that *geometrico-mathematization* is a characteristic among others that are also important?

Measuring and interpreting measurement are two different things and the latter might not be as objective as the former. In other words, there is an objective measurement and a subjective interpretation. And yet we may accept this reality easily and we may explain this phenomenon by realising that science needs metaphysics. The human being cannot be – or does not accept to be – bound and thus metaphysics can expand both our sense experience and our rational experience, and all this has consequences on the way we engage with knowledge and, last but not least, with ourselves.

The Western Ancient Geometric Tradition as a Huge Problem

As Patrick Heelan explains, “The philosophical tradition in the West began with the Greek philosopher of the sixth century B. C. E.” (Heelan 2016: 55) and this vision is commonly accepted among scholars. All that is written above departs from this tradition. The reading of the body painting was presented in terms of two and three-dimensional flat geometry refers to the Euclidean tradition which was greatly widened with topological and Riemannian geometry. If the non-Euclidean geometries came from a necessity of working in a different way in mathematics and on its necessity to fulfil requirements of new physics, it would be also important to reflect on the way how they may make sense of important achievements of human civilization in which body painting would represent an early and deep consciousness of the *self*.

The classical order in architecture is the metaphor of the human body and therefore all classical based architecture would claim to be humanized. The order – more than any other architectural element – symbolizes the *geometrico-mathematization* of architecture for it makes the *outrageous* thing of *geometrico-mathematizing* the human body on a non-realistic form and, in artistic terms, it also represents an acme of the figurative *versus* abstract phenomena – a conceptual system still valid today when referencing and conceiving art.

The rational approach to the classical temple by Abbé Marc-Antoine Laugier (1713-1769), emphasised the tectonics of the temple but by taking the classical order and confronting the charge-support system with the metaphor the human body, we are confronting the problem of data-measurement and its interpretation. [Fig. 12] Thus, changing attitudes in rationality and metaphysics have had strong consequences for art and science.

Where shall we stand? Are there some interpretations better than others? There certainly are, but it might be difficult to distinguish them. And yet there must be unavoidable *consequences from our choices*. *Reading art and architecture according to forms of geometrico-mathematization* implies decisions of choosing – especially – between geometric systems and the reasons to ground our options might be either rational or metaphysical and we may be aware that this is the background to our choices. The problem of data-measurement *versus* interpretation seems to be omnipresent in our decisions.

The western philosophic tradition has been particularly ambitious and would aspire to explain everything and anything comprehensively, a perfect absolute body of knowledge. This scientific project was accepted up to the time of Kant. “With Kant, however, the view of the human subject as a spectator of nature was supplanted by the view that in some sense the human subject constitutes the universal forms under which nature presents itself. The Kantian view attacks the scientific ideal, it denies that perfect science is capable of revealing nature independently of human presence in nature.” (Heelan 2016: 57) Consequences to the presence of mathematics in architecture must be tremendous and Husserl’s *Krisis* will present a further problem in this regard. Heisenberg’s uncertainty expresses this problem at the atomic level, but the general problem of science is general and deep.

Science, and philosophy in general, seem to need a *neo-accuracy* to overcome inaccuracy, inconsistency, and lack of meaning. And much of what represents the *Krisis* of science lies on the incapability of description given by and through a *geometrico-mathematization*. And among such approach irrational numbers appear to be the major constants of both nature and architecture, somehow more abstract and descriptive than the platonic solids. What appears to Heisenberg's uncertainty would be taken by phenomenologists as the source of accuracy itself given by the intricate mingling of *body-mind-world* and Husserl's eidetic reduction would fulfil all possible gaps that knowledge as such, e.g., science and philosophy, must not have. That is, an impressive project in the making equivalent to all dreams of creating an absolute knowledge of that kind only gods can have and understand.

Reason, Metaphor and Constants of Nature

Constants of nature look pretty much attractive. The perfect universe in numbers free from metaphysics. Cosmos was not made *deus ex machina*. The famous Einstein's formula $e=mc^2$ concurs with this believe. Einstein himself liked the idea of constants of nature and, indeed, there are constants in nature. In this sense, the irrational numbers used in the architectural western tradition have created constants in architecture which are valid to the extent in which they have been used. Their traditional interpretation has a basic background formula which is *a form of mimesis* rooted in that tradition as well. The key that links the visual and unseen nature is connected by magic formulae that gods have hidden from mortals.

Thinking about constants of nature may not be speaking of number alone but also of unbreakable properties of the physical space and there are some properties of particular important to physics, and architecture that have already been introduced. As Jacob Bronowski puts it on discussing the properties of symmetry in two- or three-dimensional space, "Well, it is not a game. It brings us face with something which is hard to remember, and that is the that we live in a special kind of space – three-dimensional, flat – and the properties of that space are unbreakable. (...) There are only certain kinds of symmetries that space can support, not only in man-made patterns, but in the regularities which nature herself imposes on her fundamental, atomic structures." (Bronowski 1973: 174)

"Their symmetries [of crystals] are imposed on them by nature of the space we live in – the three dimensions, the flatness which we live in. And no assembly of atoms can break that crucial form of nature. Like that units that compose a pattern, the atoms in a crystal are stacked in all directions. So a crystal, like a pattern, must have a shape that could extend or repeat in all directions indefinitely. That is why the faces of a crystal can only have certain shapes; they could not have anything but the symmetries in the patterns." (Bronowski 1973: 174-176)

“Thinking about these forms of pattern, exhausting in practice the possibilities of symmetries of space (at least in two dimensions), was the great achievement of Arab mathematics.” (Bronowski 1973: 176)

Bronowski elucidates the properties of the space we live in by crossing the reality of physics expressed by mathematics and geometry with the Alhambra, in Granada, a masterpiece of Arab architecture. [Fig. 30] Indeed, the problem of the properties of the space we live in is of major importance to architecture. We may remark, as John Barrow explains, that we may be taken to believe “that things always get more complicated as we go from two to three dimensions and higher. But it ain’t necessarily so. Sometimes extra dimensions just make it harder to fit things in. Geometers since Plato have recognized that something strange happens as we go from two dimensions to three dimensions. There are an infinite number of regular (equal-sided) polygons in two dimensions by only five regular three-dimensional polyhedra: the famous Platonic solids. The symmetry required to create such solids is very demanding and very few shapes can fit into three-dimensional space. With more dimensions than three, things become more restrictive still.” (Barrow 2002: 207) [Fig. 31] And this factor might explain why the use of irrational and irrational numbers as well as systems of proportions are commonly the basis of two-dimensional geometry rather than three-dimensional. Perhaps, architects and artists embodied by the western tradition or other similar one, have found more freedom in this way. If it is so, it has been a subjective choice which, nonetheless, worked upon a system of constant properties in a space those individuals were working on. Furthermore, they might have also created a rational system for themselves which created a hierarchy of reading the space as a creative source. First, we would have the two-dimensional space and then the three-dimensional space. And this assumption seems to go along the idea of the form as contour of the object developed in the Renaissance and we even find that analytic system in the way Leonardo represents explosions and whirlpools. Euclidean-like based perception and representation – flat geometry – thus shows vitality on approaching reality.

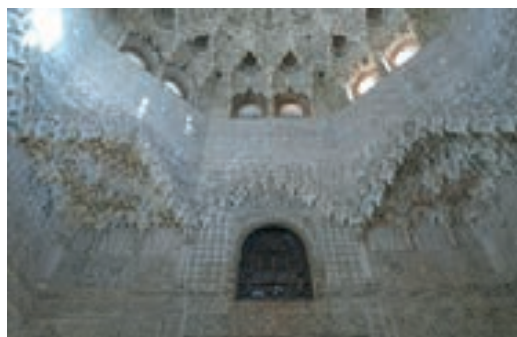


Figure 30. Sala de las Dos Hermanas (Hall of the Two Sisters), Alhambra, Granada, Spain. Built by order of Mohammed V. Ceilings carved in the 16th century (AlhambraDeGranada.org). Photography 18.08.2009 © Joaquim Santos

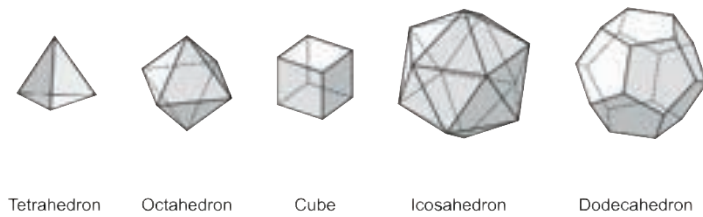


Figure 31. Platonic Solids Composition
David Eric Ffell © Wikimedia Commons
File:Platonic Solids Composition 51.svg - Wikimedia Commons

Reason, Metaphor and Constants of Nature and the Desire for a United Science

The idea of constants of nature does not stand alone but rather “radiates” to other ambitions which are not necessarily grounded in physical facts. There is an upper desire for an upper united science, a common sense that would engage the whole of ourselves with the whole we can understand. We may say that this is an idea that comes from our sense of ourselves and thus it would grant the united nature of our being. However, thinking about constants of nature we then seem to need to follow the path of natural sciences leading us to physics being the latter the ultimate authority and thus reference to all other sciences. Thus, we may question the loss of authority of other sciences, their methods, and their actual authority in providing accurate knowledge.

The problem of the method is crucial and has been largely debated and it would not be discussed along these lines, but a few questions will arise. We have referred to Ernest Nagel remarking that “we may find Euclidean language psychologically simpler than other” and we may now refer to Roger Trigg, “Science appears to be cognitively unnatural in ways in which religion, for instance, is not. Whether true or not, people find it easy to think in ways that lead to religion as distinct from the rational reflection about religious impulses, which is province of philosophy and theology. Modern science may be a demonstration of major intellectual progress, but it has not come easily, as science must, beyond appearances. Unlike religion, science, in any form at all, has not been a widespread phenomenon. It has been an achievement of human reason that goes against ordinary presuppositions. The physical world can be very surprising.” (Trigg 2015: 129-130)

Regarding the western tradition of geometric problems and its extensive use for finding the exact adequate proportions for temples we may consider that, in fact, myth and reason mingle intricately. Marguerite Neveux on criticising Ghyka on the Golden Section remarks that there is no clear border between science and myth. Thus, we simply find two different systems put together, science puts the questions and myth gives the answers. (Neveux 1995: 137, 138, 140) Yet we may classify this tendency with an ordinary tendency of mythology,

either, for a profound tendency toward metaphysics to be found in the evolution of science based on formulating questions and looking for answers. In any case, the tendency to unify knowledge from different sources might be a deep human characteristic that may result in us discounting necessary trials.

Architecture and the arts in general participate in this movement towards an encompassing knowledge but taking on Neveaux's reflection we may also proceed to fallacy. In general, this movement takes us from "subjective" aesthetics to "scientific" explanation. For instance, during the Renaissance, theory of architecture was simultaneously a theory of art as such, a theology, and a science. The idea of divine creation and its mathematical framework in cosmos meant that to discover the universe by the means of science was to approach God. As Galileo believed, God proceeds always mathematically. Without discussing the theology of the time, we can, nonetheless, quote, "To conceal a matter, this is the glory of God, to shift it thoroughly, the glory of kings." (Jerusalem Bible 1974: Proverbs 25:2) a statement that seems close to what Greeks thought of gods and *cosmos* – the visual nature given to sense experiences and the hidden nature to be found and biding both makes clear the work of gods and of God.

We make take another example from the nineteen-twentieth centuries for further analysis. The dictum *form-follows-function* that become the most representative idea of Modern Architecture combines a few beliefs. It stands as a *law* to understand any man-built artifact and the sense of being a law is a scientific conviction. Sullivan (1856-1924) in "The Tall Office Artistically Considered" refers to this law to be applied to all organic and inorganic things, "Whether it be the sweeping eagle in his flight, or the open apple- blossom, the toiling work-horse, the blithe swan, the branching oak, the winding stream at its base, the drifting clouds, over all the coursing nun, form ever follows function, and this is the law." (Sullivan 1896: 408) Thus, there is an argument regarding an overwhelming law that stems from nature and thus we may not think in *mimesis* because there is no factor of imitation, e.g., an eagle does not imitate a mountain. There is simply an inner law beyond what we see, and we must dissect it, e.g., to make our architectonic creations suitable for life.

This *dictum* then comprises the functional organization of the house but also all materials that are combined to make it a construction. Different materials imply different forms. Finally, what Sullivan proposed is to see and therefore to start from sense experience and to reflect deeply on what is hidden there, how the horse and eagle actually work, in such a way that can take us back to Leonardo's dissection of bodies of people and animals – the way how things work and what makes them possible to work in the way they do. A line of thought that we can find in the immense number of Leonardo's machinery, and in the way how new technologies and materials challenge the new form of architecture made from the iron-glass era of architecture onward.

But we must take into consideration that this form and function is taken from a Nature that Darwin has clarified and in which adaptation via form-follows-function became a new reality in natural sciences. Besides, nature could no longer be seen as static but as a system of perpetual transformations in

which energy was the factor. Atom took this idea to an acme and the idea of an elementary particle to be found across animate and inanimate nature sounds like a constant of nature. Consequently, we may observe in the Modern Architecture a tendency to participate in what we may credit as the wish for a united science being architecture, one of the sciences that converge into a magnificent body of knowledge. Le Corbusier's *Modulor* might be traced easily to the *geometrico-mathematization* of architecture following the western tradition but, in fact, there are numerous aspects that we can trace to a tendency to build a united science, or body of knowledge. (Le Corbusier 1887-1965) And a close observation of this tendency takes us to physics and to the mathematical body of support it needs. In this sense, there is a movement to avoid metaphysics on the search for a perfect explanation that is outside us, but that we have discovered. Finally, the fundamentals of Modern Architecture were considered ethical and thus we reach an environment in which morality and ethics may have – or they might be explained by – a scientific base.

Art, Science, specialization, and the demand for a united science

Being aware that the gold found on Earth was created somewhere trillions of light years away and being capable to interpret that data would not have been possible without the demand for a united science and without working from metaphysics – but this does not mean that the product of the process of inquiry becomes a metaphysical belief. Realising that we know more about our planet – e.g., investigating the geological contents of the different geological layers across the planet – by collecting data from the stars and vice-versa that we may know more about stars by observing what is right in our sight would have not been possible with that aspiration. Yet, we are confronting the fact that scientific knowledge is counterintuitive, it is not obvious from the direct experience of the world even if, or when, it derives from it.

Often it sounds difficult to combine art works from different ages. We may take several art works from across the world from different ages that enable us to create a coherent story. Most people would find this hard and yet a chronological set of events – cause-effect related – will be found easy. Yet, we would hardly imagine Picasso (1881-1973) without bringing together primitive art – e.g., the African mask and his Bull's Head (1942) made from a bicycle saddle and handlebars. [Fig. 32] Also, the different interpretations of Classical Architecture from the Renaissance up to the Post Modernism may not seem obvious since they implied to put together different ages. We cannot forecast what the artist's next work will be, but we cannot either determine with precision what the new scientific discovery will be about.



Figure 32. Pablo Picasso. Bull's Head (Tête de Taureau), 1942.

CarlaMartin04 © Wikimedia Commons

<https://commons.wikimedia.org/wiki/File:TeteDeTaureauPicasso.png>

Regarding Picasso's the Bull's Head, the chronological arrangements of objective data – for the present case art works – implies that we face the problem of interpretation of data as well the question for explanation since “Decisions about the reducibility or otherwise of levels of explanation is an intrinsically metaphysical decision.” (Trigg 2015: 120) This problem is important in science in general but in art goes from history of art as science to creativity. What moved Picasso on from knowledge patterns provided by objective data were metaphysical based decisions which were able to combine not only art but all human civilizations across the globe.

Here we may have a sense of encompassing knowledge different from that of the *form-follows-function* but rather a phenomenon equivalent to what Thomas Kuhn describes in terms of paradigms where a new one “absorbs” and enlarges previous ones – e.g., Maxwell's equations on electricity and magnetism. We are thinking about the extent to which Picasso's Art Works were able to – and were only possible by – bringing together western tradition and non-western art tradition. In general, and regarding cubist representation of faces, Cubism is about this phenomenon insofar as faces recall the expression of masks on their synthesis but from the western tradition side reality seems more complex. And the “African statuettes he had seen in the studios of Derain and Matisse” were fundamental to this process. (Miller 2001: 106)

Henri Poincaré's four-dimensional world

The hypercube is a time free geometric entity in the *four-dimensional* world and *space-time* is something else. The problem of the fourth-dimension motivated hard discussion at the turn of the nineteenth century to the twentieth century. Myth and metaphysics mingled to solve the problem of what it was and to the extent people could, or could not, be aware of this new world by the terms of

visualization. It looked far more abstract and alien to sense of experience being one of those examples where science seems counter intuitive.

The fourth dimension offered a challenge that non-Euclidean geometries did not and thus attract major speculation such as “the theosophic notation of “astral sight” on the “astral plane.” Astral vision meant seeing the true and absolute representation of an object as it occupies the astral plane, which extends out to infinity. Once situated in the astral plane, one sees all sides of an object at once, so that it will seem almost unrecognizable. Coincidentally, there is no perspective point as there is with the inferior viewing.” (Miller 2001: 104)

The fourth dimension appealed strongly to the way in which mathematics can extend our perception and sense of the world for a rational system assured that other world has to be rational and mensurable. But a set of numbers organized on – say – a polynomial equation may need an image. And, let it be an image of the world we live in, or either, of a “parallel” world, things might turn out to be complicated when we cannot simply imagine it besides a set of abstract data. Then, would mathematics really extend our senses of reality – would we need to change our conception of reality? – or is there an ultimate reality out there given by four dimensions?

Henri Poincaré could find an explanation of what that four-dimensional world would be and suggested a way of understanding it in terms of sense experiences accessible to us in the three-dimensional world we live in. As he puts it, “The images of external objects are painted on the retina, which is a plane of two dimensions; there are *perspectives*. But as the eye and the objects are movable, we see in succession different perspectives of the same body from different points of view... Well, in the same way that we draw the perspective of a three-dimensional figure on a canvas of three (or two) dimensions, so we can draw that of a four-dimensional figure from different points of view. This is only a game for the geometer. Imagine that the different perspectives of one and the same object succeed one another.” (Miller 2001: 105 apud Poincaré 1968: 92)

However, interpretations of Cubism under Picasso’s brush are hardly consentaneous with Poincaré but rather with Henri Bergson’s “l’Introduction à la métaphysique” (1903), “Consider the movement of an object in space. My perception of the motion will vary from the point of view, moving or stationary, from which I observe it”, (Bergson 1912: 105) that is, we have a three-dimensional space plus time which is deeply different from the notion of having an arithmetamization of space in four dimensions in which – and additionally – we will use a notion of time to see what this four- dimensional is about and Picasso did it by reducing those four dimension to two dimensions – and that was his genius. (Henri Bergson 1859-1941)

Being or not being motion referable seems very important for it may mean that Picasso’s references were an actual arithmetamization of space to which Poincaré provided a form of visualization which would represent the belief – also shared by Einstein – that geometry would provide a unified theory for physics – the demand for a united science – whose essence could be inquired by art means. Then, we must consider that Carlo Carrà’s “*Cavallo e cavaliere*” o “*Il*

cavaliere rosso", (1913) may be further away from Picasso and yet influenced by Bergson and Eadweard Muybridge (1830-1904) and his numerous experiments on analysing animal motion – including human motion – where we also find similarities between Marcel Duchamp's "Nude Descending a Staircase, No. 2" (1912), and Muybridge's photogravure "Descending stairs" (1884/86). [Figs. 33-34] (Carrà 1881-1966).



Figures 33, 33A (detail). Marcel Duchamp. "La boîte verte", 1934, with a reproduction of "Nude Descending a Staircase, No. 2," 1912, painting at the Philadelphia Museum of Art.

Revista Ramona © Wikimedia Commons

https://commons.wikimedia.org/wiki/File:Caja_verde_de_Marcel_Duchamp.jpg

Figure 34. Eadweard Muybridge. Nude man descending stairs, 1884/1886.

This is plate 126, captioned "Descending stairs". "Eadweard Muybridge. Animal locomotion: an electro-photographic investigation of consecutive phases of animal movements. 1872-1885. USC Digital Library, 2010." Original source: "Eadweard Muybridge. Animal locomotion: an electro-photographic investigation of consecutive phases of animal movements. 1872-1885, published under the auspices of the University of Pennsylvania. Plates. The plates printed by the Photo-Gravure Company. Philadelphia, 1887."

University of Pennsylvania © Wikimedia Commons

[https://commons.wikimedia.org/wiki/File:Nude_man_descending_stairs_\(rbm-QP301M8-1887-126\).jpg](https://commons.wikimedia.org/wiki/File:Nude_man_descending_stairs_(rbm-QP301M8-1887-126).jpg)

Yet, if we proceed with the idea of taking separately a time free, four-dimensional world on one hand and a four-dimensional *space-time* on the other, we may arrive at Husserl's notion of number which is contentious to pure mathematics and which is not only space-free but also psychologically free. (Patras 2020: 162-163) Thus, we may consider that what made Picasso's Cubism possible is a perfect mathematical framework event independent from the realm of physics where time is unavoidable. A perfect symbiosis of mathematics and art.

Conclusion

Meaning and significance of a *mathematical-like* character in architecture is mutable. Besides the constructive character of architecture as a material thing which involves precise acquaintance with the laws of physics such as the mechanics of surfaces (e.g., mortars that grant different parts to stick together, or one block on another), attrition or gravity, other forms in which mathematics is present in architecture seem to be exclusively of a subjective choice of the architect. Even complex forms that may arise from contemporary developments on mathematics which make the use of algorithms possible and its inherent calculus possible, are of a subjective choice.

We may agree that the idea of extending our senses through mathematics operates in a way that it extends our imagination. At the core of mathematics, phenomena like this witness the humanizing character of architecture since it represents a unique human creation of its own –human creation *par excellence*. Yet, there are two “mathematical entities” that were not specifically discussed here and thus not combined with all that was analysed, and they are the *Zero* and *Infinity*.

The western tradition can bound our understanding of a wide presence of geometry in civilization all across the world since the dawn of civilization. Yet, it is also true, that it was the western tradition of algebra and geometry that has given us a clear understanding of non-Euclidean geometries to understand what lies beyond traditional Euclidean geometry.

The four-dimensional space represented a challenge for Euclidean and non-Euclidean geometries and time is fundamental to distinguish perfect mathematical entities to physical entities represented by mathematical means. Thus, Cubism needs to be considered in a double-crossing manifold of art and physics, and art and mathematics and conclusions would not overlap but rather, will reveal interesting aspects about the overwhelming equation of the construction of a united science in respect of specifics rather than from being obsessed with a unifying constant.

The idea of constants of nature is fundamental but it goes far beyond a Euclidean understanding of the world. However, the idea of the invariable is certainly there as expressed by definitions, postulates, and axioms. This idea represents the believe that there is something there outside our mind that is unchangeable. However, Kant introduced the idea of our interference with phenomena westudy. Heisenberg, Plank, Husserl, and the other phenomenologists

have then worked on their own ideas about what acquaintance with phenomena may mean in terms of a reliable knowledge.

There is the dream for a united science and constants of nature appeal strongly to this idea. However, such a dream does not stem from any physical evidence but rather from a metaphysical desire that, nonetheless, seems of major importance to knowledge development. The problem of interpretation of data is crucial in this “twist”. But we must consider that this path takes us either to fallacy or to accuracy.

The idea of a united science tends to take us to physics and to its mathematical framework and thus to impose on other sciences its own methods in what is extremely critical to evaluate art and architecture by making them depend on either quantitative data, or scientific methodology that works insofar as the product of a specific science. Method can thus bias the object under study. Perhaps we can continue reflecting on Max Plank’s words,

“Science cannot solve the ultimate mystery of nature. And that is because, in the last analysis, we ourselves are part of the mystery that we are trying to solve”, Max Plank. (Barrow 2002: 23 apud Wilber 1985: 153)

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