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HEURISTICS IN INVENTORY CONTROL: OPERATIONAL EMPIRICAL EVIDENCE

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Abstract: This paper intends to be a methodological contribution to the comparison of results obtained inventory management through formal modeling and those implemented through simulation and optimization processes, particularly using genetic algorithms (evolutionary). The development focuses primarily on backorder situation for models of reorder point, periodic review and (S, s, R) where the demand per unit time and lead time are normal distributions. The analyses are realized for a set of heuristics applied to the management of a durable fast moving item. The main limitations in modeling and optimization of numerical nature are refereed.

Key-words: Heuristics, simulation, optimization, evolutionary algorithms, reorder point policy, periodic review policy, (S, s, R) policy.

1. Introduction

The importance of policies adjusted to inventory control are well known, being extensive the bibliography dealing more or less this particular subject. The contributions made by Hadley (1963), Silver (1998), Zipkin (2000) and more recently Muckstadt (2010) were the background for sustained and broad range of developments. It can be considered extremely practical in applications and on the other the work of mathematical nature more or less abstract without immediate prospect. Models, by their nature, are representations or abstractions of real operating environments, perhaps not capturing all the factors involved or possibly pondering them in a less adjusted basis. This work intends to be a methodological contribution to the comparison between the results obtained through simulation particularly using genetic algorithms (evolutionary) and optimization analytic processes. The development focuses primarily on backorder situations for models of reorder point, periodic review and (S, s, R) with normal distribution to the demand per unit time and lead times. The paper begins with a brief description of the main heuristics most commonly used in inventory control, highlighting the main assumptions introduced during the modelling process. In a second point the most relevant characteristics in the simulation optimization process are described, in general the genetic algorithms.

In a third point the scenario and the input parameters for the empirical work are established, being presented the results for numerical optimization. In a fourth point, policies of reorder point (s, Q), periodic review (S, R) and (S, s, R) are modelled in ExtendSim® environment. The importance of the Optimizer block, the operating conditions and the length of the simulation run periods are described. Finally, in a fifth point, the most relevant results are presented and synthesizing the most important conclusions.

This paper describes part of the research undertaken in this specific context and which supports the current developments with concrete applications to the area of Supply Chain Management, Lopes (2014).

2. Heuristics in inventory control

Policies for inventory control in a situation of continuous demand can generally be classified into three broad categories: That of reorder point, periodic review and with mix characteristics (S, s, R) integrating particularities of the previous two. Each policy has advantages and disadvantages under the practical point of view, which may lead the manager to choose preferably by one, Lopes (2014). Out as most relevant, among others, the size of buffer stocks, the possibility of consolidation of orders, more or less difficulty in operation, etc.

2.1 Model of reorder point

This policy is based on the fact that an order is placed for Q units when the stock on hand reaches the order point s. The graphical representation of its operation is shown in Figure 1.

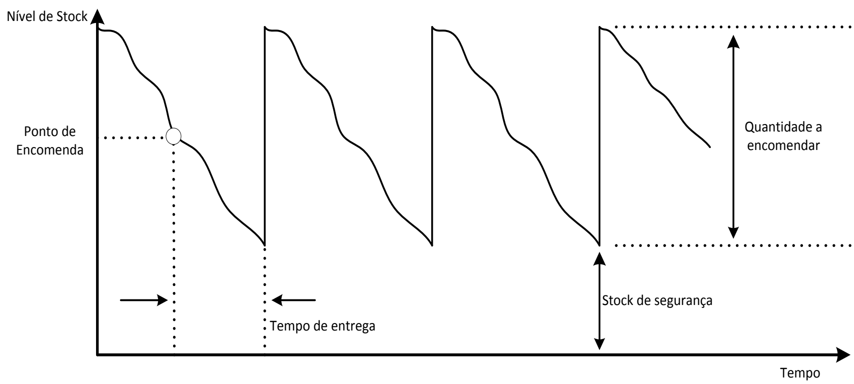


Figura 1 - Reorder Point Policy

By admitting that the distribution of demand per unit time and lead times are normal and that the modelling parameters are defined by:

v → Unit variable cost of the item;
A → Fixed ordering cost;
B_2v → Cost per unit short;
r → Carrying charge;
D → Average demand per unit time;
σ_D → Demand per unit of time standard deviation;
L → Average replenishment lead time;
σ → Lead time standard deviation.

It is possible to obtain the cost per unit cycle time:

$$Kt = \frac{AD}{Q} + (OH)vr + \frac{B_2 vD(BO)}{Q} \quad (1)$$

where OH is the average amount of stock on hand and BO the average number of backorder per cycle.

Note that OH can be expressed as a function of inventory stock position by

OH = Inventory average stock position (PS) - Average stock on order (SE) – BO (2), but:

$$PS = \frac{1}{2}[(Q + s) + s] \quad (3)$$

and:

$$SE = Q + s - (s - \hat{x}_L + Q) \quad (4)$$

where $\hat{x}_L = LD$ is the mean demand during the lead time. Then:

$$(OH) = \frac{Q}{2} + s - \hat{x}_L + (BO) \quad (5).$$

Note, as mentioned by Muckstadt (2010) that only one backorder per unit time is admitted. The short units per BO can be determined by:

$$\int_s^\infty (x - s) f(x) dx \quad (6)$$

where f(x) is the distribution of demand during the lead time, with mean and standard deviation respectively equal to:

$$\hat{x}_L = LD \ (7) \ \text{ e } \ \sigma_L = \sqrt{L\sigma_D^2 + \sigma^2 D^2} \ (8).$$

But, according to Silver (1998):

$$\int_s^\infty (x - s) f(x) dx = \sigma_L G_u(K) \quad (9)$$

where:

$$G_u(K) = \int_k^{\infty} (u - k) \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du \quad (10)$$

with:

$$K = \frac{s - \hat{x}_L}{\sigma_L}$$

Thus are obtained

$$K_t = \frac{AD}{Q} + \left[\frac{Q}{2} + s - \hat{x}_L + \sigma_L G_u(k) \right] vr + \frac{B_2 vD}{Q} \sigma_L G_u(k) \quad (11).$$

Another approach is to assume that

$$E(OH) \approx \frac{Q}{2} + s - \hat{x}_L \quad (12)$$

Whereas the average backorder quantity is very small relative to the average stock on hand and therefore negligible.

The expression that results in , will be

$$K_t = \frac{AD}{Q} + \left[\frac{Q}{2} + s - \hat{x}_L \right] vr + \frac{B_2 vD}{Q} \sigma_L G_u(k) \quad (13)$$

It should also be noted that in many practical situations the values set for the policy of reorder point admit for the calculation of s that:

$$s = \hat{x}_L - k\,\sigma_L \quad (14)$$

where k is an empirical value given by the standard cumulative normal distribution. Q is in turn determined by the expressions minimizing costs in deterministic demand models situations or through any other approaches.

2.2 Periodic review model

The periodic review policy is based on the observation of the stock on hand of R in R time units (review period), ordering the amount needed to achieve a level pre-specified S. The graphical representation of the policy is shown in Figure 2.

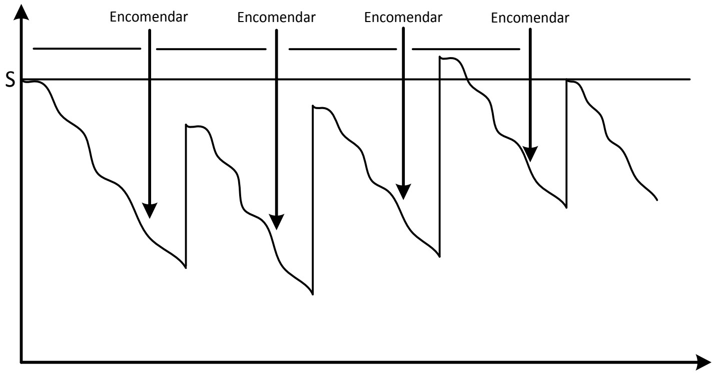


Figura 2 - Periodic Review Policy

When considering the conditions set out above for the previous case and that M is the fixed cost of ordering and review, R is the review period

$$\hat{x}_{L+R} = (L + R)D \ (15) \quad \text{and} \quad \sigma_{L+R} = \sqrt{(L+R)\sigma_D^2 + \sigma^2 D^2} \ (16)$$

Where $\hat{x}_{L+R}$ is the average demand during the lead time and review period and σ_{L+R} is the standard deviation of demand in the same period. It is then possible to obtain the expression of the cost per unit cycle time:

$$K_t = \frac{M}{R} + \left[\frac{DR}{2} + S - \hat{x}_{L+R} + \sigma_{L+R} G_u(k)\right] vr + \frac{B_2 v}{R} \sigma_{L+R} G_u(k) \ (17)$$

When considering the conditions to those observed in the reorder point it is possible to obtain:

$$K_t = \frac{M}{R} + \left[\frac{DR}{2} + S - \hat{x}_{L+R}\right] vr + \frac{B_2 v}{R} \sigma_{L+R} G_u(k) \ (18)$$

It is important to note that modelling in any of the policies (reorder point or periodic review) assumes that there are no crossing orders, getting the order in the sequence which they are realized.

As in the previous case it is usual to use a value of S computed by:

$$S = \hat{x}_{L+R} + k\,\sigma_{L+R} \ (19)$$

being the value of R determined by several methods.

2.3 Model (S,s,R):

This model is a hybrid of reorder point and the periodic review. In this case, the stock on hand is observed every R units of time, in case it lies between two pre-specified parameters s and S is not performed any order, if lower s order to achieve the level S. The policy operation is shown in Figure 3.

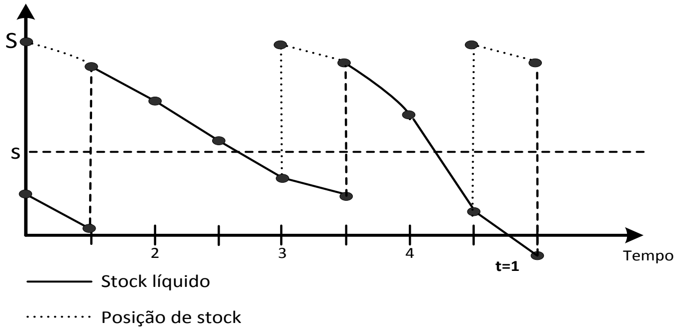


Figure 3 - Operation of the policy (S,s,R)

The simultaneous determination of the three parameters is referred by Silver (1998) and Muckstadt (2010) as complex, suggesting the use of heuristics as the best suited process. The procedure developed by Wagner (1975) is recommended in this case and it will be briefly described in the following paragraph.

2.4 Numerical optimization
2.4.1 Reorder point model

By differentiating equation (11) in order to the variables Q and s can be obtained by considering that:

$$\frac{\partial K_t}{\partial Q} = \frac{\partial K_t}{\partial s} = 0,$$

$$Q = \sqrt{\frac{2D\left[A + \sigma_L B_2 v G_u (k)\right]}{vr}}$$

and:

$$p_{u\geq}(k) = \frac{Qr}{rQ + B_2 D} \quad (21)$$

Where is the complement of the cumulative function of the standard normal distribution. The optimum values for Q and s are found successively iterating between (20) and (21) in order to obtain a degree of adjusted precision. The suggested approach is identical to the previous procedure, but now when using equation (13) an expression similar to (20) is obtained, coming to:

$$\partial K_t / \partial s = 0 \text{ a probabilidade } p_{u\geq}(k) = \frac{Qr}{B_2 D} \quad (22)$$

As reported by Muckstadt (2010), the convergence of the algorithm in both situations is verified, because Kt is convex in the area that contains the optimal solution for most distributions of lead time demand.

2.4.2 Periodic review model

In this particular case there follows a different procedure given the complexity presented to $\partial K_t/\partial R$. In this circumstance, it is preferable to use equally spaced R values for the calculation of Kt (through (17)) for a pre-specified S obtained using the expression considering that $\partial K_t/\partial S = 0$:

$$p_{u\geq}(k) = \frac{rR}{rR + B_2} \quad (23)$$

Thus electing the values of R and S that lead to a minimum . A second approach would be using the expression (18) and admitting that:

$$p_{u\geq}(k) = \frac{rR}{B_2} \quad (24)$$

The conditions of convexity of K_t are verified in accordance with the observation considered for the reorder point.

2.4.3 Model (S,s,R):

The heuristic developed by Wagner (1975) can be described for a given pre-set revision period through the following steps:

1. Determinar $Q = \sqrt{2AD/vr}$

2. Calcular u, tal que: $G(u) = \dfrac{vrQ}{B_2 v\sigma_D\sqrt{L+1}}$

3. Se Q>1.5D, considerar:

$$s = (L+1)D + u\sigma_D\sqrt{L+1} \quad (25)$$

$$S = s + Q \quad (26)$$

otherwise go to 4.

4. Determine v, such that: $\Phi(v) = \dfrac{B_2}{B_2 + r}$

Onde $\Phi(v)$ is the cumulative function of the standard normal distribution for the argument v.

Admitting that:

$W = min\ (u,v)$

$s = (L+1)D + W\sigma_D\sqrt{L+1}$ (27)

$S = (L+1)D + \min\{u\sigma_D\sqrt{L+1} + Q;\ v\sigma_D\sqrt{L+1}\}$ (28)

3. Optimization in simulation

The optimization applied to simulation is particularly complex for a number of reasons which are referred for example by Banks (2010) and Law (2007).

- The results of simulation models are random variables, so we can speak only in probability of choosing an optimal selection of input parameters. In principle this situation may be overcome by performing a high number of runs for each tested solution, significantly reducing the variance of the result.
- The algorithms to be used shall ensure that they are inherently asymptotically consistent, i.e., it is possible to achieve an approximation of the optimal value as the number of runs increase. Beyond this fundamental property its operational structure will contain search strategies associated to the random component of the situation.

Currently, as referred by Fu (2002), the simulation optimization processes can generally be broken down into two phases:

- Generation of candidate solutions;
- Evaluation of solutions.

In fact, the processes of optimization software packages developed in the simulation are based almost entirely on meta heuristics and predominantly evolutionary algorithms (genetic), that iterate a family of solutions rather than a single point, even incorporating some memory in structure. A new solution (new offspring) is achieved by randomly modifying individuals from the parent population. This operation is usually called mutation materialized by the addition of reduced normal random variables. In some situations recombination (crossover) is employed so that two parents are combined to generate a new solution, for example, by selecting the first half of the first parent factors and the second half of the second. The election of parents as reported by Buchholz (2005) is implemented in those presenting high probability of better performance and obtained with low computational effort. Assessment is doing through a specific objective function.

The statistical process of searching and selection of the solutions is based in most cases on the Rinott (1978) procedure, which is developed in two phases.

An indifference-zone parameter d*>0 is previously defined, such that the decision maker does not care to choose the solution k - 1 if the means μ_k and μ_{k-1} verify $\mu_k - \mu_{k-1} < d^*$. It is admitted that $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_k$.

The process ensures that the best element is selected with a predetermined probability P*, with $1/k < P^* < 1$ since $\mu_k - \mu_{k-1} < d^*$.

Consider then n_0 as the sample size of each solution. In the first phase sample means $X_i(n_0)$ and variances $s_i^2(n_0)$ are calculated such that

$$X_i(n_0) = \frac{1}{n_0} \sum_{i=1}^{n_0} X_{ij} \quad (29) \quad \text{and} \quad s_i^2(n_0) = \frac{1}{n_0} \sum_{i=1}^{n_0} (X_{ij} - X_i(n_0))^2 \text{ para } i = 1,\ldots,k \quad (30).$$

Based on the initial number of replications and the sample variances $s_i^2(n_0)$ obtained in the first phase, the number of additional simulation replications for each individual in second stage is N_i - n_0 with

$$Ni = \max\left\{n_0, \left[\left(\frac{h}{d^*}\right)^2 s_i^2(n_0)\right]\right\} \quad (31).$$

Where $h=h(k,P^*,n_0)$ is a constant which solves the Rinott integral.

From the results of the first and second phases new means are determined, such that

$$X_i = \frac{1}{N_i}\sum_{j=1}^{n_i} X_{ij} \quad (32)$$

being elected the solution with higher (lower) mean.

This procedure was subsequently modified in order to make it more efficient on the computational point of view, Lopes (2014), see for example the contributions made by Boesel (2003) and Buchholz (2005). For a state of art about this subject is recommended Lee (2013).

4. Empirical scenarios, modelling parameters and numerical optimization

The empirical scenario baseline uses a durable item with a unit price of 100 monetary units (m.u)/unit (v), with a demand for unit of time (day) of 220 units/day (D) and a standard deviation of 28 units (σ_D). The lead time identically follows a normal distribution with mean 5 days (L) and standard deviation 1 day (σ). The fixed order cost was estimated in 3 m.u/order. The annual carrying charge (r) has an average value in range referred by Muckstadt (2010), 22.5 %. The estimated cost of stockout per short unit was approached by the methodology described by Anderson (2006) which admits an impact in the short and medium term not only in selling the product but in future orders. Used the average results obtained by the authors for a sample of 13816 items, and adjusted the price for the product in question (B_2v=0,29 m.u./short unit). In the case of review cost, the value was taken to 3.1 m.u a little higher than A. Table 1 summarizes the parameter values to be used in empirical scenario.

Table 1
Parameters of Empirical Scenario

Item	Daily mean demand (D)	220
	Standard deviation of daily demand (σ_D)	28
	Lead time mean (L)	5
	Lead time standard deviation (σ)	1

Costs (u.m)	Unitário (v)	100
	Fixed ordering (A)	3
	Review + Order (M)	3.1
	Daily Holding (vr)	0.062
	Stockout (B_2v)	0.29

Regarding the numerical optimization the steps were followed as described in 2.4.1, 2.4.2 and 2.4.3, yielding the results shown in Table 2. An additional set of testing scenarios is further defined and clarified as well as the numeric equations used to calculate the parameters of each of the policies.

Table 2
Empirical Scenarios

Scenario Characteristics	Parametric Determination (Equations)	s	Q	R	S
PE1	(20);(21)	1283	279	-	-
PE2	(20);(22)	1183	368	-	-
PE3	(14);(20)	1393	209	-	-
PE4	Q = √(2AD/vr); (21)	1348	146	-	-
RC1	(17);(23)	-	-	2.9	1668
RC2	(18);(24)	-	-	2.2	1567
RC3	Q = √(2MD/vr); R=Q/D; (19)	-	-	0.67	1543
RC4	Q = √(2MD/vr); R=Q/D; (23)	-	-	0.67	1409
M1	R(RC1)* - Wagner algorithm	1313	-	2.9	1384
M2	R(RC3) - Wagner algorithm	1313	-	0.67	1384
M3	R(RC1); s=S-Q(PE1); S(RC1)	1389	-	2.9	1668

*The value of R used was that employed in RC1 scenario, etc.

5. Simulation Models

The simulation models of the three policies were developed in ExtendSim8® environment, with the structures shown in Figures 4, 5 and 6.

Figure 4 - Simulation Model for Policy (s,Q)

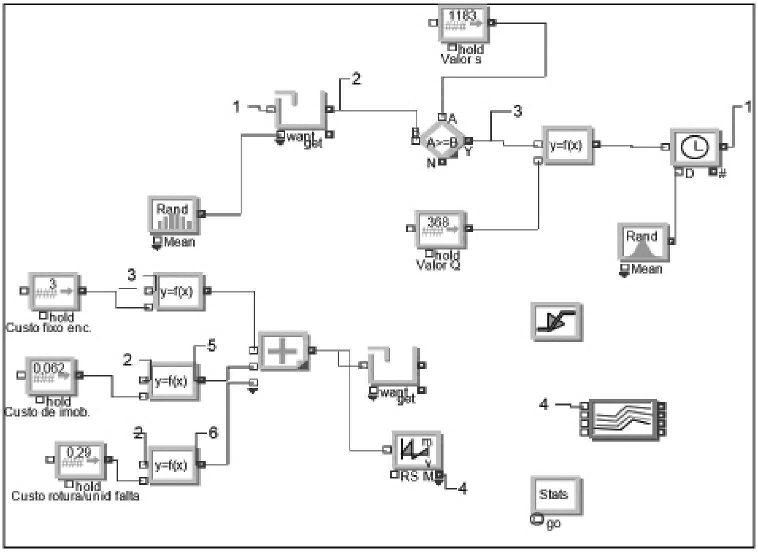


Figure 5 - Simulation Model for Policy (R,S)

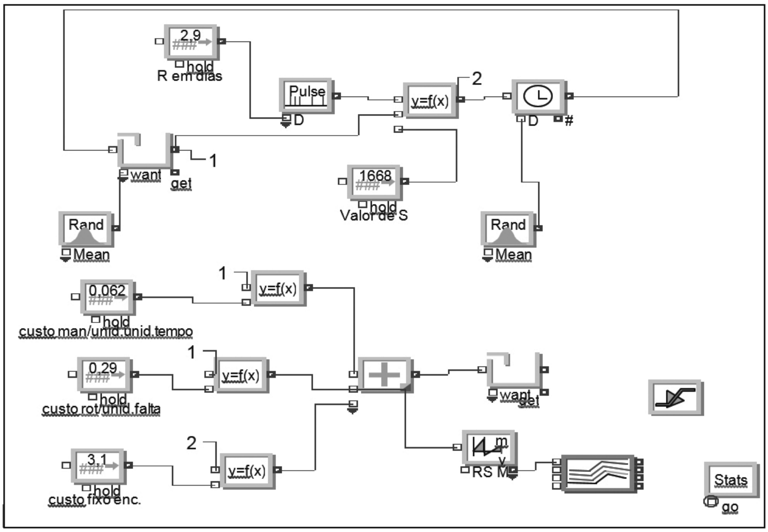
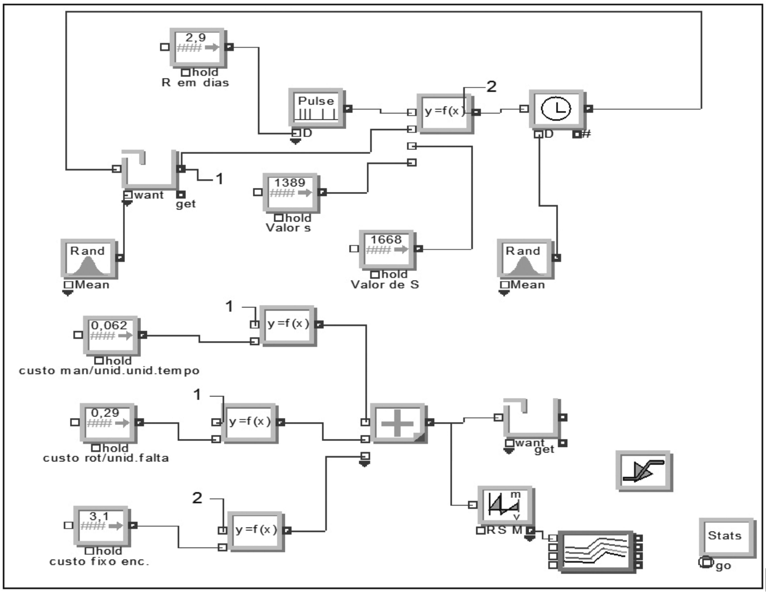


Figure 6 - Simulation Model for Policy (S,s,R)



The general operating characteristics and features of each block can be found in ExtendSim User Guide (2007). It is however relevant to mention the following singularities:

- Only blocks of libraries Value and Plotter were used given the specificity of continuous simulation applied;
- There was no particular care on using sample sizes of smaller size, since the computational time is not a restrictive element during the empirical experimentation. This allowed to largely neutralize the initial transient conditions;
- In all structures the Statistics (Stats) block was introduced allowing exporting the results to an EXCEL spreadsheet facilitating the subsequent statistical treatment (in Excel or SPSS).

The Optimizer block was introduced in each structure. It uses an evolutionary algorithm (genetic) with characteristics similar to those described in Section 3 as referred by Zvirgzdina (2013). Amplitude ranges of variation were established for any of the parameters, large enough to present no restriction on determining the best solution in each case. Note, however, that imposed in the case of model (S, s, R) the condition S ≥ s. The criterion used was the minimization of the total average daily cost (order / revision + order, holding and stockout). Still admitted a dimension of population of solutions size 10, with 100 the maximum number of replications to consider a solution (the block starts the optimization process using one sample only, this was incremented in subsequent generations, until the maximum). The optimization concluded after analyzing 1000 generations. The procedure and parametric optimization for each policy was repeated 20 times and for each of the best solutions found was materialized 30 replications with a size of 104 days for which the total mean value was calculated. This procedure followed by selecting the five best average results by performing 60 replications each with 104 days. The election progressed through this procedure for sets of 3 and 2 solutions. Finally the one with the lowest total mean value was selected. Note that a direct comparison of means is acceptable given the low variance of their estimators, taking into account not only the large number of replications but also the size of each one. This is in fact the same principle followed by Rinott (1978). For the three policies characterized by the parameters given in Table 2, 120 replications of the cited same size were made.

6. Analyses of Results

As mentioned in point 4, 20 experiments for each of the policies in evaluation (reorder point, periodic review and (S, s, R)) using the evolutionary algorithm included in the Optimizer block were made. Table 3 summarizes the main characteristics of the solutions.

Table 3 - Optimization Using the Evolutionary Algorithm

Policy	Solution mean value	Solution Standard deviation	Convergence (%)	
			Máximum	Minimum
(Q,s)	57.21	1.58	99.44	94.30
(R,S)	41.90	1.11	99.55	98.21
(S,s,R)	51.84	8.79	99.95	97.25

We notice that the solutions are closed, as either the standard deviation or the degree of convergence are consistent indicators of this circumstance. Note that the degree of convergence measures the relative variation (among the population of the top ten elected solutions) between the maximum and the minimum of average total cost of each policy. The policy (S, s, R) presented, however a higher variability, a fact that is not strange given the larger number of parameters to be estimated by the algorithm. This peculiarity is salient in Figures 6, 7 and 8 where is shown the variability of the estimates of the parameters in these experiments.

Figure 6 - Variation in Parametric Optimization / Policy (Q,s)

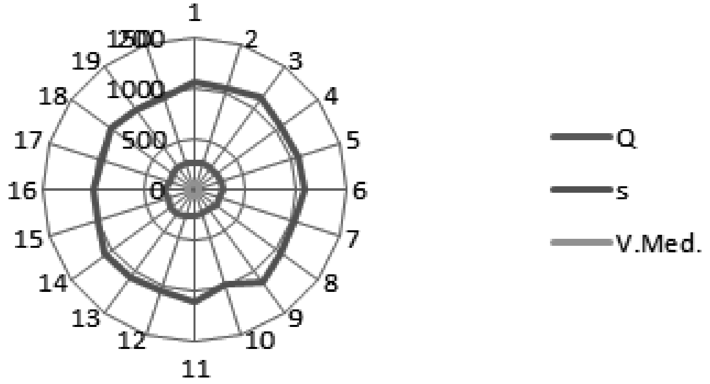


Figure 7 - Variation in Parametric Optimization / Politica (R,S)

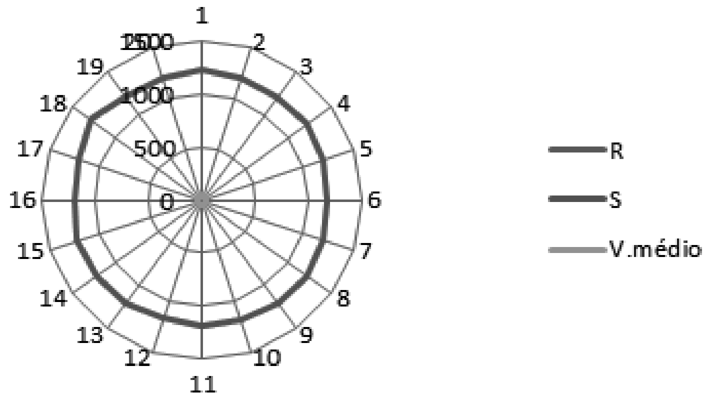
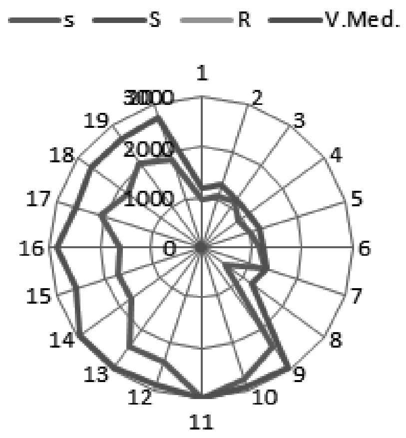


Figure 8 - Variation in Parametric Optimization / Politica (S,s,R)



In the first two policies there is a marked stability, as noted; while in (S, s, R) s and S parameters show significant changes for minor variations on mean total unit cost. Then proceeded to the election of the parameters that had lowest average cost in accordance with referenced in Section 4. This results are shown in Table 4.

Table 4 - Parametric Optimization via Simulation

Policy	Parameters				Daily Mean Cost (Last selection)
	Q	s	S	R	
(Q,s)	245	1132	-	-	49.58
(S,R)	-	-	1197	4	37.44
(S,s,R)	-	913	1168	4	42.36

It should be noted the numerical approximation of the parameters (S, s, R) and those observed for (s, Q) and (S, R) given the natural hybrid characteristics of that policy. In order to statistically compare the performance of each of the policies tests t Student for differences of means for each model was done (120 runs for periods of 104 days). The results are shown in Table 5.

Table 5 - Student's t test for Policies Difference of Means

Pair	Confidence interval (95%)		Linear Correlation Pair
	Upper Limit	Lower Limit	
(Q,s)-(R,S)	12.39	7.44	0.03
(Q,s)-(S,s,R)	10.77	4.81	-0.10
(R,S)-(S,s,R)	0.75	-4.99	0.08

When observing the table we can infer that the policies of periodic review (S, R) and (S, s, R) outperform the (Q,s), not being possible however to establish a statistical degree of preference between policies (R,S) and (S, s, R). Heuristics with the parameters specified in Table 2 were simulated as stated through 120 runs each with duration of 104 days. The results for each heuristic were compared by Student's t test. Analyzing these results it can be concluded the best performance of the heuristics PE1, PE2 and M1 being followed by RC1, RC2 and M3. The remaining results show much lower performance and even PE3 and PE4 do not resist to transient initial conditions being in permanent loss. These results are most easily observed using cluster analysis. The method of hierarchical aggregation of Ward (1963), and the Euclidean distance were applied, verifying an adjusted dimension for a set of 6 clusters. This result is shown in Table 6.

Table 6 - Heuristics Means Cluster Analysis

	PE1	PE2	PE3	PE4	RC1	RC2	RC3	RC4	M1	M2	M3
	1	1	2	3	4	4	5	6	1	6	4
Mean Value Total Cost (diary)	55.3	68.66	16347	107591	89.4	118.41	228.24	220	76.67	220.91	91.02

Note that the heuristics are presenting more elaborate theoretical approaches that lead to better results, verifying that the estimates supported by most empirical procedures lead to significantly higher costs, it should be emphasized that the heuristics lead to results different from those observed for optimization, since the best policy is not identical in both cases. Thus it is important to observe the estimated average costs for stockout and holding for the three heuristics that comprise the cluster 1 and 4, those that come from the optimization and obtained through the numerical approximations. In this circumstance 120 runs of size 104 days were performed for each situation. The results obtained are shown in Table 7.

Table 7 - Stockout and Holding Costs

Policy	Solution	Parameter	Simulation Approach		Numerical approach	
			Stockout costs	Holding costs	Stockout costs	Holding costs
Reorder point (Q,s)	Obtained by Evolutionary Algorithm	Q=245 s=1132	13.08	34.14	24.89	64.33
	Obtained Numerically (PE1)	Q=279 s=1283	6.12	48.05	6.29	21.70
Periodic review (R,S)	Obtained by Evolutionary Algorithm	R=4 S=1197	21.42	20.06	56.68	27.21
	Obtained Numerically (RC1)	R=2.9 S=1668	9.10	75.80	13.10	23.56
(S,s,R)	Obtained by Evolutionary Algorithm	R=4 s=913 S=1168	23.04	16.78		
	Obtained Numerically (M1)	R=2.9 s=1313 S=1384	23.24	52.79		

Analyzing the results of Table 7 it appears to be realized that the simulation results for the parametric estimates obtained via numerical and evolutionary algorithm for the holding cost are invariably significantly higher than on the second case, which leads to the inference that models based on heuristics lead to conservative policies with great part of stock without rotation. This has an obvious result, for the particular cases of political (Q, s) and (R, S), lower stock out costs. This condition is however less apparent in politics (S, s, R). The result may find support when considering, in numerical approximations, the existence of a backorder per cycle and perhaps the non-existence of cross orders, which will significantly reduce the variance of demand during the lead time, as stated Bischak (2013). It is interesting to notice, for PE1 and RC1 heuristics, the closeness of estimates of the stock-out costs obtained for the numerical solution when using simulation and that achieved by the formal model, thereby allowing concluding the necessity of a more consistent modelling for the inventory holding costs.

6. Conclusions

The work presents a methodological basis for comparing the performance of heuristics in the specific context of inventory management. The empirical basis of experimentation corresponded to a durable and fast moving item, with mean particularities of cost, demand and lead time. In this circumstance interests synthesize the main conclusions:

- Evolutionary algorithms are particularly adapted in simulation optimization, leading to a consistent set of solutions particularly in what convergence concern. For all three variants tested there is a higher stability of the estimated parameters to the policies (Q, S) and (R, S), finding an increase in variability for the (S, s, R) option. This fact naturally infers that the stability of parameter estimation in simulation, in the specific context of inventory management, is reduced with some meaning as far as is enlarged the number of parameters to be estimated.
- That for an item in the above specified conditions and preferably using as criteria the cost per unit cycle time is statistically possible to infer that the periodic review policy and (S, s, R) have a higher performance then (Q, s). The preference, however, between models (R, S) and (S, s, R) could not statistical be established.
- The empirical work that led to eleven heuristics described earlier is clear, that those with more consistent theoretical supports led to better results. The most empirical procedures led to significantly higher costs and often are divergent with the initial transient conditions. This fact indicates in this case the need for a finer adjustment (tuning) of the parameters, possibly via simulation.
- The selection of policies using the estimators obtained numerically and those obtained through simulation optimization are diverse. The situation is due to formal models overestimate the operational parameters leading to conservative policies with excess stock without rotation. This fact is evident as a result, especially for policies (Q, s) and (R, S), where the stockout costs are lower. This finding is justified in considering the numerical modelling, the approach of having a backorder per cycle or a nonexistence of crossing orders which overestimates the holding costs. This allows us to conclude the

necessity of more consistent modelling for these costs.

- The alternative to numerical estimate parameters of inventory management is the use of complementary techniques of simulation and optimization as presented in this work, with obvious cost benefits of the policy performance.

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